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Physical substantiation of an opportunity of artificial change of body weight

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Abstract

Evangelical legends about «walking on waters» though they are apparently a fruit of imagination, can have a natural physical substantiation. Change of weight of accelerated moving bodies, confirmed with laboratory measurements of temperature dependence of body weight, in combination with an assumption of non-stationary character of the gravitational field of the Earth, directly leads to the conclusion regarding an opportunity of appreciable artificial change of body weight. The simple phenomenological model is described, according to which at the certain phase ratio of vertical oscillations of a trial body and small (with a relative level of amplitude equal to the tenth - 100-th fractions of per cent) own periodic fluctuations of normal acceleration of free falling (AFF) there are possible both a significant increase and a reduction of average weight of such a body. It is shown that at frequencies of the vertical fluctuations essentially exceeding frequency of own fluctuations of AFF the effect of reduction of average body weight prevails. Results of an experiment with "instant" measurements of acceleration of free falling of a mechanical rotor with horizontal axis of rotations which have confirmed the periodic changes of rotor AFF followed from the specified model are given. A good outlook for development of physics of gravitation and development of new principles of movement, set-up of precision experiments with weighing of bodies moving with acceleration, including those oscillating vertically along trial bodies, is noted.

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Keywords: gravitation, weight, acceleration, phenomenological model, rotor.

NOMENCLATURE

- A_c coefficient of interaction of elastic and gravity forces by counter of \vec{a} and total vector of gravity force ($m^{-1} \cdot s^2$)
- A_p coefficient of interaction of elastic and gravity forces by passing of \vec{a} and total vector of gravity force ($m^{-1} \cdot s^2$)
- \vec{a} acceleration vector of external force (value of vector, $m \cdot s^{-2}$)
- $a(t)$ variable acceleration of the material point ($m \cdot s^{-2}$)
- B amplitude of oscillation (m)
- F frequency (s^{-1})
- $f(x)$ frequency function (#)
- g_0 normal acceleration of gravity ($m \cdot s^{-2}$)
- m mass (kg)
- T average absolute temperature of air (K)
- V volume of air in the container (cm^3)
- v velocity of a sound ($m \cdot s^{-1}$)
- x ω / Ω (#)
- β relative amplitude changes of AFF (#)
- γ relative temperature change of piezoelectric ceramics weight by 1 degree (K^{-1})
- $\tilde{\gamma}$ calculated factor ($kg^{-1/2} \cdot m^{5/2} \cdot s^{-1}$)
- $\Delta\bar{g}$ average change of acceleration of free falling ($m \cdot s^{-2}$)
- $\Delta\vec{g}_c, \Delta\vec{g}_p$ increments of acceleration of gravity (value of vectors, $m \cdot s^{-2}$)
- $\Delta\tilde{T}$ average change of temperature of air in the container (K)
- $\delta\tilde{m}$ change of weight of the container (kg)
- θ the phase of changes of AFF (rad)
- μ A_c / A_p (#)
- ρ density ($kg \cdot m^{-3}$)
- τ period of oscillations (s)
- Ω frequency of changes of AFF ($rad \cdot s^{-1}$)
- ω angular velocity of oscillations of test body - material point ($rad \cdot s^{-1}$)

INTRODUCTION

In [1-3] the gravitational analogy of Faradays' electromagnetic induction phenomenon is considered.

Accelerated under action of external (for example, elastic) forces, the movement of a test body downwards causes the increment $\Delta\vec{g}_p$ of acceleration of the gravity, acting on a body and directed from the centre of the Earth. On the contrary, the accelerated movement of a test body upwards is accompanied by a value $\Delta\vec{g}_c$ increase of acceleration of the gravity acting on the body. Change of acceleration of the gravity acting on a body, moving with acceleration \vec{a} under influence of the elastic force, in the elementary (linear) approximation, is represented as

$$\Delta\vec{g}_{p,c} = -\frac{\vec{g}_0}{|\vec{g}_0|}(\vec{g}_0 \cdot \vec{a})A_{p,c} \quad , \quad (1)$$

where symbols p,c mean passing (p) and a contrary (c), in relation to a direction of vector \vec{g}_0 of normal acceleration of a gravity, orientation of a vertical projection of vector \vec{a} of acceleration of external forces, and factors A_p and A_c characterize a degree of change of values $\Delta\vec{g}_{p,c}$.

1. WEIGHT OF MECHANICAL OSCILLATOR

If the massive body (for example, a ball) under action of the external, electromagnetic in nature, elastic force makes harmonious oscillations along a vertical with frequency ω and amplitude B , the average for the period $\tau = 2\pi / \omega$ of fluctuations value $\Delta\vec{g}$ of change of acceleration of free falling (AFF) of such mechanical oscillator is equal to the sum of average changes of AFF in movement of a body passing and contrary to vector \vec{g}_0 ,

$$\Delta\vec{g} = \Delta\vec{g}_p + \Delta\vec{g}_c \quad (2)$$

and at constant $g_0 = |\vec{g}_0|$ it is equal

$$\Delta\vec{g} = -\frac{g_0 B \omega^2}{\pi} (A_p - A_c). \quad (3)$$

From 3, it follows that at $A_p \succ A_c$, the average acceleration of free falling of mechanical oscillator, for example, a rotor with a horizontal axis of rotation, is less than value g_0 of normal acceleration of the gravity force. The reduction, averaged on several series of the measurements of the apparent weight of a rotor with horizontal axis, was observed in experiment [4], by results of which for the material of a rotor (stainless steel) it is possible to approximately estimate the order of value of difference $(A_p - A_c) \approx 10^{-7} g_0^{-1}$.

The absolute values of factors A_p and A_c can be measured on the basis of the shock mechanical experiments accompanied by the high, above $10^5 ms^{-2}$ accelerations of interacting bodies. For steel samples the order of values A_p and A_c is approximately equal to $10^{-2} g_0^{-1}$ [1,5].

2. TEMPERATURE DEPENDENCE OF WEIGHT OF BODIES

If to examine, as the considered above test body, a microparticle of a solid state body linked by elastic

forces of inter-atomic interaction with other similar particles, then formulas 1-3 allow to explain the influence of temperature on acceleration of free falling (weight) of such a body.

The problem of influence of bodies' temperature on the force of their gravitational interaction has been discussed since long ago and the first precision experiments in this field were already carried out at the beginning of the XXth century [6]. The next stage of experimental studies of the said specified problem fell to the beginning of the current millenium when in Russia there were published the results of laboratory measurements of temperature dependence of weight of metal bars, indicating an appreciable negative temperature dependence of the gravitation forces [2,7,8], recently these results were confirmed in works of Chinese scientists [9].

The physical substantiation of relatively strong influence of temperature on force of gravitation consists in deep interrelation of electromagnetic and gravitational interactions, and their dependence on the accelerated movement of the microparticles forming a massive body, with intensity growing with growth of temperature [1,3]. In experiments [7,9], the weighed samples were heated up to comparatively high temperatures - from ten degrees up to hundreds.

A possible, in such conditions, influence on results of measurements of the thermal air convection, the change of temperature of the scales mechanism, the thermal change of residual magnetization and adsorption of moisture on the surface of samples, and so on – naturally caused caution and even mistrust in estimations of the obtained results. Meanwhile, the results of weighing the heated metal samples were obtained at high enough levels of an effective signal to noise ratio, with the careful account for the influence of the mentioned factors.

In the described experiment, there was carried out the weighing of samples of PZT-piezoelectric ceramics, whose temperature increased by near 2°C in respect to the normal room temperature (24°C). In so doing, the influence of temperature factors on accuracy of measurements of weight of samples was reduced to a minimum.

The design of the weighed container is shown in Fig. 1.

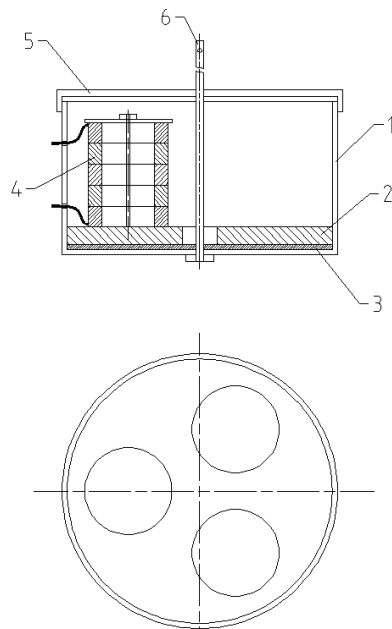


Fig. 1. The arrangement of container. 1- body, 2 – base, 3 – laying, 4 – PZT-pile, 5 – cover, 6 – hanging

bar.

The container was placed in the closed box of analytical scales, the high-frequency electric signal was fed to electrodes of piezoelectric ceramics by means of elastic copper conductors $85 \mu\text{m}$ in diameter and 150 mm in length. The weighed sample is made in form of three "piles" ("sandwiches") of parallel-connected piezoelectric ceramic rings, 5 rings in each "pile", fixed on the massive brass base; the external diameter of rings is 22 mm , the internal diameter is 16 mm , height is 6 mm ; the full weight of 15 rings is equal to 112.9 g . In parallel to the power supply terminals of piezoelectric ceramics, there was connected the variable inductance for adjustment of resonance frequency of the supplied signal equal to 389 kHz , which allows to achieve the most effective heating of samples; the amplitude of the resonance signal is equal to 40 V . The readout of scales was carried out by the elongation method with the period of scale beam oscillations equal to 19.7 s . At full weight of the container equal to about 470 g , the error in reading out the changes of weight in time did not exceed 30 mcg .

An example of typical experimental time dependence of the container weight change is shown in Fig. 2.

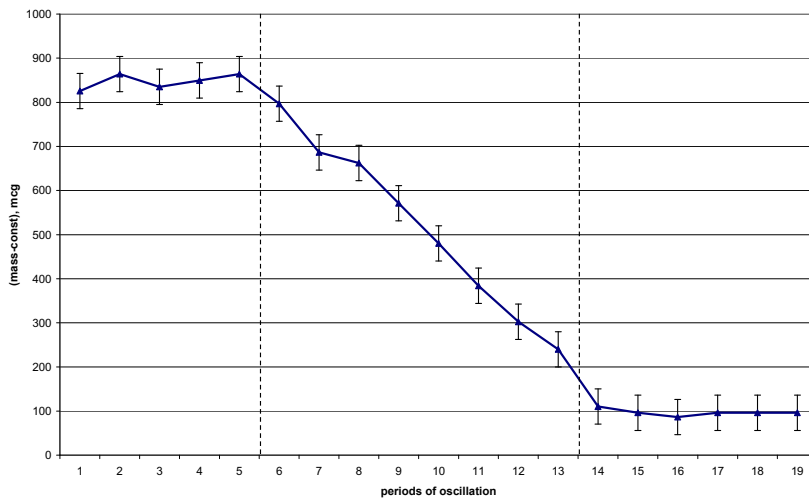


Fig. 2. Experimental time-dependence of container mass by heating PZT-pile from 24.0 till 25.7°C . Touch lines is "in" and "out" moments. 1 period = 19.7 s .

The temperature of walls of the container remained practically a constant. On Fig. 3 the results of measurements of temperature of PZT-pile and air in the top (most heated) part of volume of the container are given.

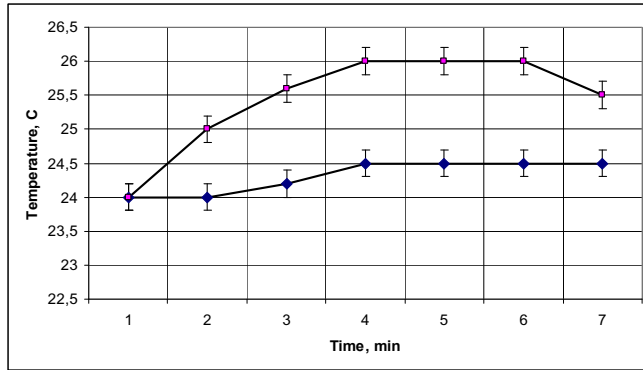


Fig. 3. An example the dependence of temperature of PZT-pile (the upper curve) and temperature of air in the top part of the container (the under curve) from time of heating. The point 1 on time-axis corresponds to the moment “in” of signal, a point 4 – “out”.

Change $\delta\tilde{m}$ of weight of the container, caused by temperature change of density of air taking place in it, equally

$$\delta\tilde{m} = \rho V \Delta\tilde{T} / T \quad (4)$$

where ρ is density of air (1.19 kg/m^3), V - volume of air in the container (150 cm^3), T - average absolute temperature of air (297 K), $\Delta\tilde{T}$ - average change of temperature of air in the container. It according with Fig 3, for duration of heating near 2.7 min, $\Delta\tilde{T} \approx 0.25 \text{ K}$ and the corresponding change of weight of the container $\delta\tilde{m} \approx 150 \text{ mcg}$, that much less than full temperature change of weight of the container (700 mcg , Fig. 2).

According to Fig. 2, for $\Delta T = 1.75 \text{ K}$ and $\Delta m = 550 \text{ mcg}$, the relative temperature change γ of piezoelectric ceramics weight by 1 degree,

$$\gamma = \left(\frac{\Delta m}{m} \right) \frac{1}{\Delta T} \quad (5)$$

is equal to $\gamma \approx -2.8 \cdot 10^{-6} \text{ K}^{-1}$.

In [3,7] it is shown, that, in classical approximation,

$$\gamma \propto \frac{v}{\sqrt{\rho}} = \tilde{\gamma} \quad (6)$$

where v - velocity of a sound in a sample, ρ - density of a material.

Experimental values of γ [7] and calculated sizes of factor $\tilde{\gamma}$ are given in the Table.

Table. Characteristics of Samples and Results of Measurement of γ and Calculate values of $\tilde{\gamma}$

Sample	Lead	Copper*	Brass**	Titanium	Duralumin	PZT
$\rho, \cdot 10^3 \text{ (kg} \cdot \text{m}^{-3})$		11.34	8.89	8.55	4.50	2.79

7.20					
$v, \cdot 10^3 (m \cdot s^{-1})$	2.64	3.80	3.45	5.07	5.20
3.50					
$\tilde{\gamma}, (kg^{-1/2} \cdot m^{5/2} \cdot s^{-1})$	0.783	1.275	1.181	2.391	3.114
1.306					
$\gamma, \cdot 10^{-6} (K^{-1})$	4.56	6.50	4.50	8.70	11.60
2.8					

(*) – twist sample, (***) – measured in Dewar

Their magnitudes normalized on the maximal value (for duralumin) are shown in the Fig. 4.

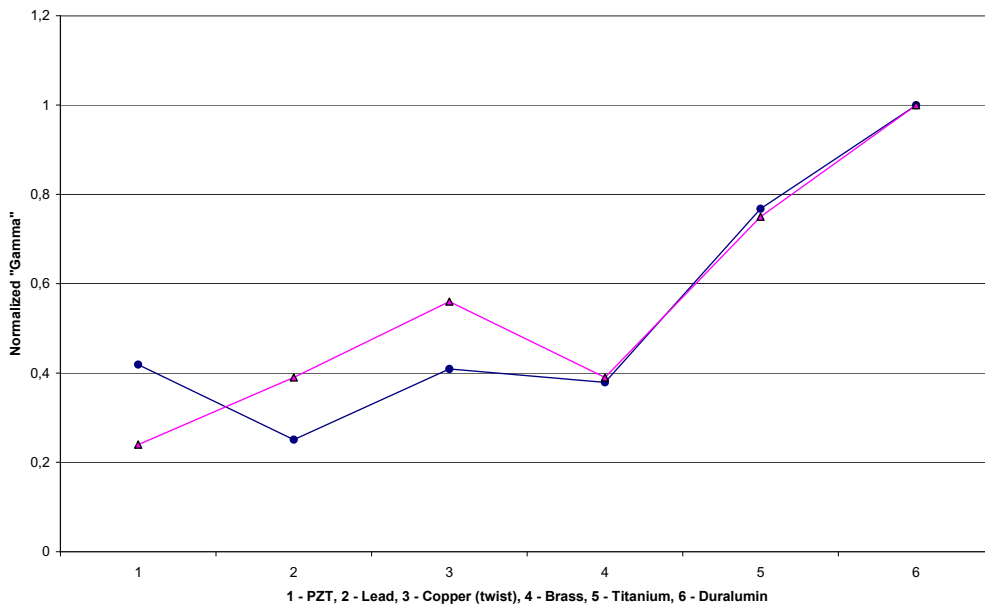


Fig. 4. Calculate $\tilde{\gamma}$ (●) and experimental γ (Δ) normalized values of "gamma"

Let's note that close conformity of γ measurement results is realized with essentially different dimensions and configurations of the samples and containers which were used.

The satisfactory conformity of calculated and experimental data proves the correctness of physical preconditions put in a basis of elementary classical model of temperature dependence of weight [2,7]. It is necessary to note, that outside of classic approximation 6, by near to absolute zero temperatures of a weighed sample, the temperature dependence of weight of bodies, apparently, has other character and is not so strongly expressed, as at normal temperatures [10].

So, the laboratory experimental data, obtained in heating of piezoelectric ceramic samples for near 2 °C, confirm the negative temperature dependence of such sample weights. These data will essentially agree with high-temperature measurements of weight of non-magnetic metal bars.

3. WEIGHT OF OSCILLATOR IN A VARIABLE FIELD OF GRAVITATION

The considered above elementary model can be formally generalized, having introduced the time variable $g_0(t)$ value of normal acceleration of the gravity. Modern ballistic gravimeters provide the precision measurements of absolute values of g_0 , thus the best results have been obtained in statistical processing of the thousands of the given selected measurements of acceleration of free falling (AFF) and long access times of measurements (from seconds to days) [11,12]. Obviously, with such measurement techniques, the high-speed, having time of relaxation less than 0.1 s, fluctuations of value g_0 essentially can not be registered. Meanwhile, in view of the complex physical processes occurring in the core and volume of the Earth, and also under influence of external astronomical factors, the presence of rather strongly expressed maxima in a high-frequency (for example, a range of several hundreds – thousands of Hz) spectrum of fluctuations of value g_0 is probable. Following such assumption, we shall present elementary time dependence $g_0(t)$ as

$$g_0(t) = g_0(1 + \beta \sin(\Omega t + \theta)) \quad (7)$$

where Ω – frequency of changes of AFF value, β - their relative amplitude, θ - the phase. Acceleration $a(t)$ of the material point making harmonious oscillations along a vertical with amplitude B is equal to

$$a(t) = B\omega^2 \sin \omega t \quad (8)$$

where ω - frequency of fluctuations.

The averages for oscillation half-cycle $\tau/2$ of values of changes of accelerations $\Delta\bar{g}_p$ and $\Delta\bar{g}_c$ are equal to

$$\Delta\bar{g}_p = -A_p g_0 B \omega^2 \frac{2}{\tau} \int_0^{\tau/2} \sin \omega t (1 + \beta \sin(\Omega t + \theta)) dt \quad , \quad (9)$$

$$\Delta\bar{g}_c = -A_c g_0 B \omega^2 \frac{2}{\tau} \int_{\tau/2}^{\tau} \sin \omega t (1 + \beta \sin(\Omega t + \theta)) dt \quad . \quad (10)$$

The relative change of AFF of the oscillator, in view of 2, shall be presented as

$$\frac{\Delta\bar{g}}{g_0} = 4\pi A_p B F^2 f(x) \quad , \quad (11)$$

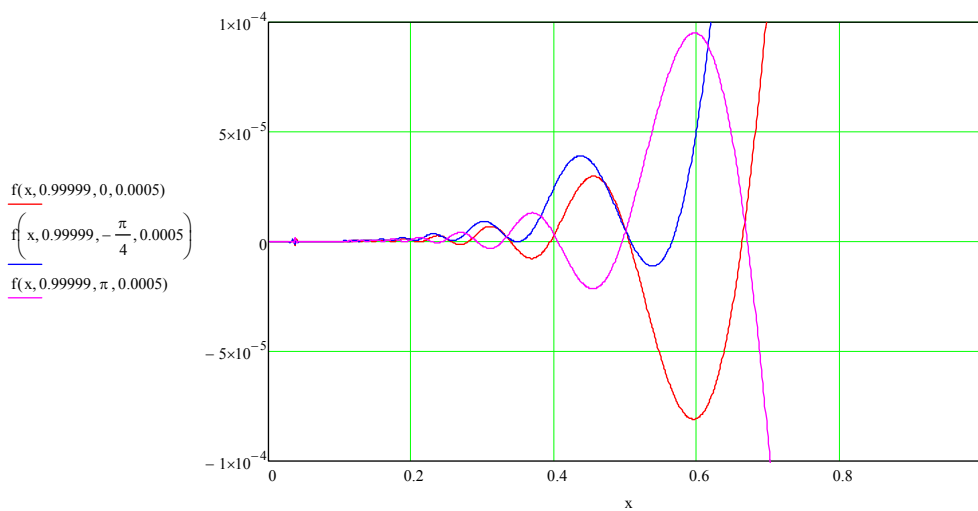
where $F = \Omega/2\pi$, $x = \omega/\Omega$ and frequency function $f(x)$ is equal to

$$f(x) = -x^2 \left[\int_0^{\pi} \sin z (1 + \beta \sin(xz + \theta)) dz + \mu \int_{\pi}^{2\pi} \sin z (1 + \beta \sin(xz + \theta)) dz \right] ; \quad (12)$$

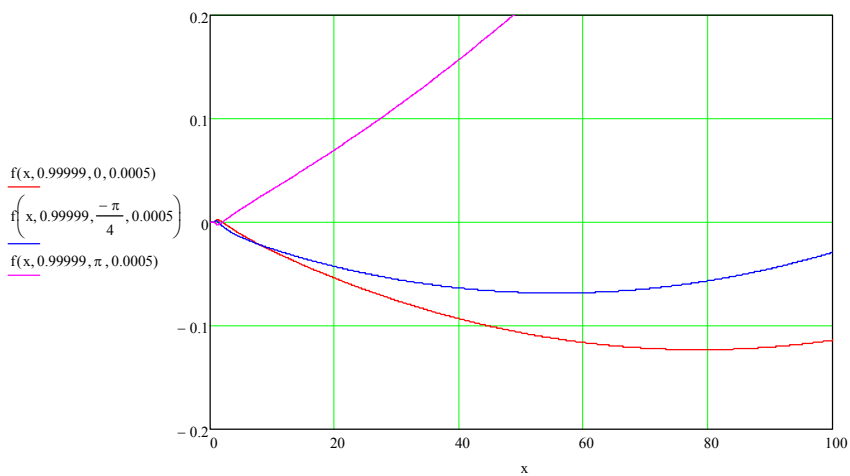
here $\mu = A_c / A_p$ and $z = \omega t$.

Examples of frequency functions $f(x, \mu, \theta, \beta)$ at various parameters μ, θ, β , and both low (a) and

high (b) values of x are shown in Fig. 5, 6.



a.



b.

Fig. 5. Examples of frequency functions $f(x, \mu, \theta, \beta)$ at low (a) and high (b) values of argument x ; relative amplitude of fluctuations AFF $\beta = 0.0005$.

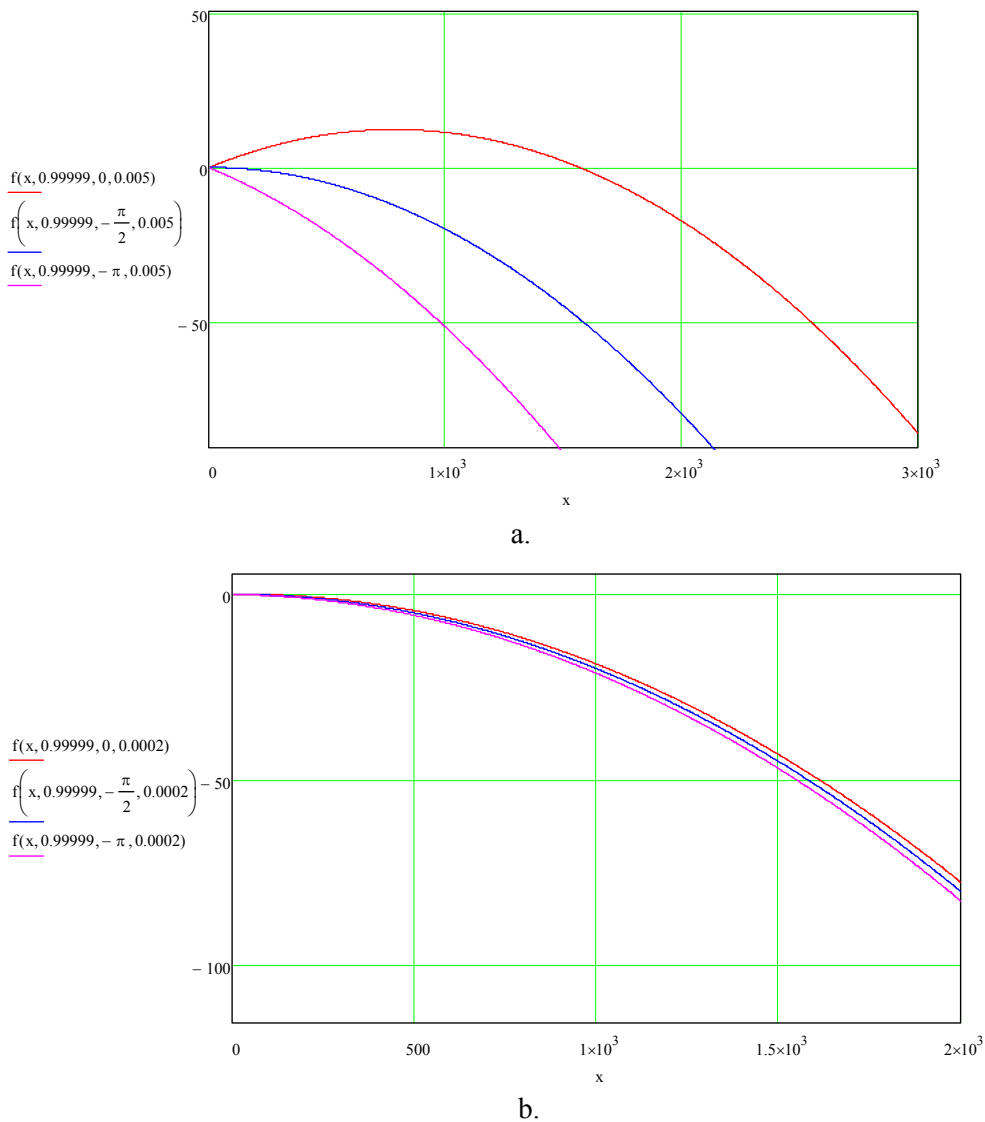


Fig. 6. Examples of frequency functions $f(x, \mu, \theta, \beta)$ at the high values of argument x ;
 a. - relative amplitude of fluctuations AFF $\beta = 0.005$, b. - $\beta = 0.0002$.

Obviously, the sign and a general view of functions $f(x)$ essentially depend on parameters μ, θ, β . According to estimations given above, in the calculations, $\mu = 0.99999$ is assumed. The given calculated dependences show that even at small, with relative value of about the 100-th fractions of percent, amplitudes β of fluctuations in value of normal acceleration of the gravity of the Earth, the weight of mechanical oscillator can be changed appreciably.

At frequencies ω of oscillations, with an order of the frequency Ω of own fluctuations of AFF, in area $x \leq 1$, the weight of oscillator is periodically changes with frequency, with sign and values of such changes essentially depending on a difference of phases θ of oscillations and AFF (Fig. 5. a). At high ($x \gg 1$) frequencies of oscillator, the monotonous dependence of average weight of oscillator on frequency of its fluctuations is taking place, with influence of phase θ being insignificant (Fig. 6. b). Such reduction of weight of oscillator at high frequencies of fluctuations will agree with temperature dependence of weight of bodies as the frequencies of thermal fluctuations of microparticles of solid state bodies are rather high and lie in the field of the hypersound [13].

4. EXPERIMENTAL DEPENDENCE OF ACCELERATION OF FREE FALLING ROTOR

Experimental check of the dependence of average weight of the above-considered oscillator on frequency of its fluctuations can be executed, measuring the instant values of acceleration of free falling rotor. Mechanical rotor is the system of the accelerated, moving on a circular trajectory microparticles, forming a solid state body, and linked to each other by forces of elasticity. With horizontal orientation of rotor rotation axis, the vertical component of trajectories of movement of particles of the rotor corresponds to the oscillations of such particles considered in item 3. Measurements of instant values of free falling acceleration of the closed container with the rotor of vacuum mechanical gyroscope fixed inside are described in [14,15]. The rotor (mass is 250 g) gathered momentum up to the maximal frequency 400 Hz, then during the run out time (about 22 min) its frequency smoothly decreased, the container was periodically dropped down, and by method of falling scale, the instant values of acceleration of free falling of the container were measured. The example the frequency dependence of change of acceleration of free falling container with a rotor fixed inside it, and with horizontally positioned axis of rotor is shown in Fig. 7.

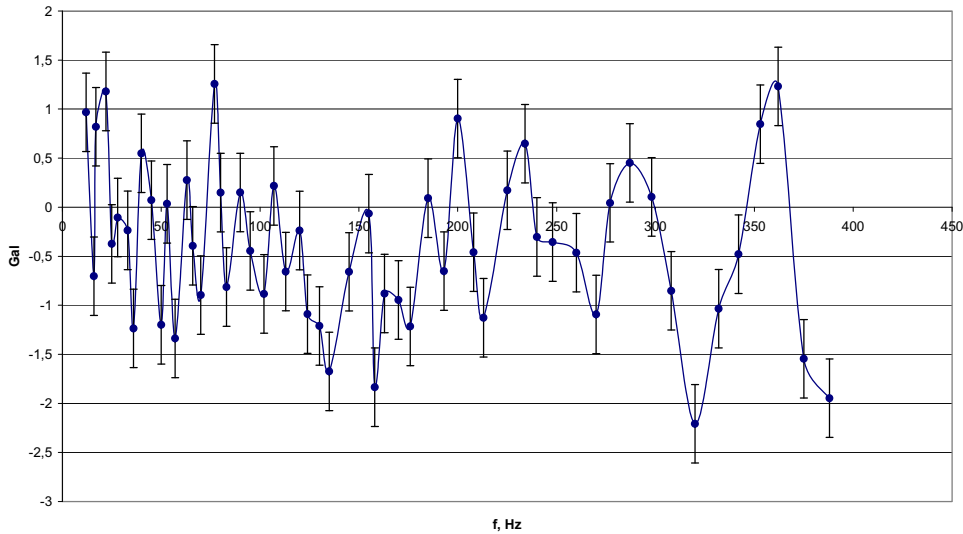


Fig. 7. The frequency dependence of free falling acceleration of the container with horizontally positioned rotor; the changes of AFF (*Gal*) relatively to the value of AFF with the stopped rotor have been shown.

Comparing Fig. 5.a and Fig. 7, it can be seen that the area of steady periodic changes of AFF in Fig. 7 in a band of frequencies 200-400 *Hz* approximately corresponds to the area in a vicinity of value $x \approx 0.5$ in Fig. 5.a. Having substituted in 11 the experimental value $\Delta g / g_0 \sim 10^{-3}$, assuming $A_p \sim 10^{-2} g_0^{-1}$, $f(x) \sim 10^{-5}$, we obtained an estimation of amplitude $B \sim 1.4\text{cm}$ of oscillator. The given size almost coincides with radius of the rotor used in experiments. At oscillation frequencies tens times higher than the frequencies F of own fluctuations of normal acceleration of the gravity (according to the given estimations, $F \sim 300/0.5 = 600\text{Hz}$) and following the suggested model, there is observed a monotonous frequency dependence of change $\Delta \bar{g}$ of average value of acceleration of free falling oscillator, with sign $\Delta \bar{g}$ being directly determined by the difference of phases θ of fluctuations AFF and oscillator (Fig. 5.b, Fig. 6.a). Within the limits of applicability of formulas 7,11 there are possible both substantial growth and reduction of the average gravity working on mechanical oscillator on the part of the variable gravitational field of the Earth. Let's note that the independent measurements of high-frequency, in the range of hundreds – thousands of *Hz*, spectra of fluctuations of acceleration of the gravity of the Earth, executed, for example, with use of superconducting gravimeters, will allow to define modes of the matched fluctuations of oscillator at which the changes of its average weight can essentially surpass the ones described by formulas 7-11.

The above-given estimations have the selective, illustrative character. Nevertheless, the considered simple phenomenological model finely explains the experimental dependences and agrees with the known data of measurements of weight of accelerated moving test bodies.

Experimental researches into free falling mechanical oscillators (rotors, vibrators) will allow to bring the necessary specifications into the offered models, to determine the borders of their applicability, and to

prove more strictly the size parameters introduced into these models. Such researches will promote obtaining the new data on dynamic characteristics and specific features of the gravitational field of the Earth.

CONCLUSION

The considered above model does not contradict to the known experiments for exact measurements of masse and weight of bodies, and explains the influence of temperature and accelerated (oscillatory or rotary) movements of a body on its average weight. The experimental researches of gravitational analogies of the electro-dynamic phenomena should promote the active development of both physics of gravitation and its applications in metrology of weight and gravimetry.

In the immediate prospects, the following directions of researches into features of gravitational interaction of accelerated moving bodies seem to be expedient.

First, the researches into temperature dependence of weight of bodies of various physical and chemical structures, conducted in the wide range of absolute temperatures of test bodies. Second, the exact measurements of weight of bodies in a condition of oscillatory and rotary movements, and also in shock mechanical experiments. Third, the experimental researches into high-frequency ranges up to several hundreds - thousands *Hz*, spectra of fluctuations of normal acceleration of the gravity of the Earth.

The experimental results obtained during such researches will allow to specify and improve the phenomenological models in the description of the "non-classical" gravitational phenomena, breaking the frameworks of the simple Newton approximation, and probably to specify the ways of their effective practical applications.

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References

- [1] Dmitriev, A. L., On the Influence of External Elastic (Electromagnetic) Forces on the Gravity, *Russian Physics Journal*, 2001; **44** (12), 1323-1327.
- [2] Dmitriev, A. L., Measurements of the Influence of Acceleration and Temperature of Bodies on their Weight, in proceedings of *Space Technology and Application International Forum (STAIF-2008)*, edited by M. El-Genk, AIP Conference Proceedings **969**, New York, 2008, pp. 1163-1169.
- [3] Dmitriev, A. L. Analogue of Lenz's Rule in Phenomenological Gravitation, in proceedings of *Space, Propulsion & Energy Sciences International Forum (SPESIF-2009)*, edited by G. A. Robertson, AIP Conference Proceedings **1103**, New York, 2009, pp. 345 – 351.
- [4] Dmitriev, A. L., and Snegov V. S., Weighing of a Mechanical Gyroscope with Horizontal and Vertical Orientations of the Spin Axis, *Measurement Techniques*, 2001; **44**(8), 831-833.
- [5] Dmitriev, A. L., Inequality of the Coefficients of Restitution for Vertical and Horizontal Quasielastic Impacts of a Ball Against a Massive Plate, *International Applied Mechanics*, 2002; **38** (6), 747 – 749.
- [6] Shaw P. E. and Davy N. The Effect of Temperature on the Gravitative Attraction, *Phys. Rev.* 1923; **21** (6), 680-691.
- [7] Dmitriev, A. L., Nikushchenko, E. M., and Snegov, V. S., Influence of the Temperature of a Body on its Weight, *Measurement Techniques*, 2003; **46** (2), 115 – 120.
- [8] Dmitriev A. L. Experimental Study of Gravity Force Temperature Dependence, *18th International Conference on General Relativity and Gravitation (GRG18)*, Abstract Book, 2007; 77-78.
- [9] Liangzao Fan, Jinsong Feng and Wuqing Liu An Experimental Discovery about Gravitational Force Changes in Materials due to Temperature Variation, *Engineering Sciences*, China, 2010; **12** (2), 9-11.

- [10] Tajmar M., Plesescu F. and Seifert B., Measuring the dependence of weight on temperature in the low-temperature regime using a magnetic suspension balance, *Meas. Sci. Technol.* 2010; **21**, 015111 (7pp).
- [11] Torge W. *Gravimetry*, Walter de Gruyter, Berlin-New York, 1989.
- [12] Jentzsch G. et al. (Editors), Time Varying Gravimetry (Special Issue), *Journal of Geodynamics*, **38**(3-5), (2004).
- [13] Anselm A. I. *Osnovy statisticheskoy fiziki I termodinamiki*, Nauka, Moskva, 424 p., 1973.
- [14] Dmitriev A. L., Nikushchenko E. M. and Bulgakova S. A. Nonzero Result of Measurement of Acceleration of Free Falling Gyroscope with the Horizontal Axis, [http://arXiv.org:0907.2790v1\[physics.gen-ph\]](http://arXiv.org:0907.2790v1[physics.gen-ph]), pdf 2009.
- [15] Dmitriev A. L. Frequency Dependence of Rotor's Free Falling Acceleration and Inequality of Inertial and Gravity Masses, [http://arXiv.org:1101.4678v1 \[physics.gen-ph\]](http://arXiv.org:1101.4678v1[physics.gen-ph]), pdf 2011.