Paul Gerber, "Die Fortpflanzungsgeschwindigkeit der Gravitation," *Annalen der Physik*, Vol. 52, s. 415-444 (1917).

Translated from German by Walter Rella

THE PROPAGATION VELOCITY OF GRAVITY

By Paul G e r b e r † [1])

I.

In volume 43 of the J. for Math. & Phys. (Zeitschrift f. Mathematik und Physik), pp 93-104, I have shown: If it is assumed that the hitherto unexplained advance by 41`` per century of Mercury's perihelion is caused by the delay of time spent for the spatial propagation of gravity, it follows that this value equals the velocity of light, of thermal radiation and of electric waves. Attention has to be paid to what can, on the one hand, really be proven by computation and observation and what, on the other hand, is presumed in the first place without any proof. If the gravity between two masses is transmitted from the first body to the second and back again with some lag of time, one finds that this necessarily gives rise to an advance of the planets perihelion. It is, however, impossible to prove that the actual value of the perihelions advance, although it could not be deduced from disturbances of any type, could not have another origin than the presumed time lag. If this presumed origin gave a value for the propagation velocity of gravity different from the velocity of light, this would have no further meaning. Just the coincidence of both velocities vindicates this presumption and, hence, the notion of a finite propagation velocity of gravity.

This coincidence, yet, is not only the new but also the unexpected result of my derivation. Because, although it was believed, from the beginning, that the velocity of gravity would reveal itself equal to the velocity of light, measurements carried out previously have yielded a much higher value. It even seemed as if there was no time lag at all, when two masses came to attract each other. Already, in the fourth edition of his famous treatise on the development of the mechanic, M a c h drew attention to the conflicting results between my inquiry and older ones. What is the reason why this result was now obtained, which had been considered unlikely for so long? In my previous treatise I sketched the answer superficially in order to avoid extensive methodological and similar discussions. One would however comply with me that a more detailed account on this point should be given.

The following must be born in mind. Having obtained so different results for the speed of gravity by different inquiries, differences amounting to between three fifth to ten millions of the speed of light, it must be suspected that this was not due to the kind of computation or to the choice of observations but rather to the underlying assumptions giving rise to these large differences. This will be confirmed in the following. It would be premature in any case to assign, as it happens, astronomical reliability to these results. My inquiry has shown that certain notions concerning the time lag during the propagation of gravity will lead to the value of the speed of light. Therefore, the question is not if the speed of gravity manifests itself in the motions of the planets or the revolution of the moon or any other cosmic process, but, instead, to what extent fundamental ideas can be developed and substantiated concerning the question what propagates between masses, which are the parameters influenced thereby and of which type and size this particular influence is. As will be shown, older methods do not meet these challenges. I refrain, for the main, from reiterating topics of my former treatise but will rather engage in critical and historical aspects of the fundamental questions which, intentionally, have been dealt with only shortly or passed over at all so far.

[1]) This work of G e r b e r has been published as a programmatic treatise of the municipal high school at Stargard, Pommern (1902), having appeared formerly as an abstract in the Journal of Mathematics and Physics, Vol. 43, 93-104 (1898). The reprint of this programmatic treatise in the Annals meets a desideratum advanced to me from various sides on occasion of my article in these Annals, Vol. 51: 119-124 (1916). E. G e h r k e

II.

Three main points have to be put ahead. The first one concerns the concept of space wherein gravitic processes are taking place with masses at rest or in motion. The second one addresses the role played for the present question by the gravitic potential. The third one involves the lag of time – being different from the time spent for the spatial propagation - consumed for the transmittance of the gravitic action to the masses.

In pondering the riddle of gravity we usually try to find a mechanical cause giving rise to the gravity of masses. The idea of a kick and of momentum is applied frequently as hidden or overt background to help explain the successive propagation of the gravitic action. It is, however, neither necessary nor possible to do anything more than inquire, if the gravitic processes are indeed endowed with properties making them different from any other physical processes or if this exceptionality is more apparent than real. Hence, the question is to conceive gravitic processes in the context of the entire physical system. The demonstration of its successive propagation in space is the first condition which has to be fulfilled if its exceptionality were to be invalidated. Because it makes a big difference, if the gravitic phenomenon is perceived from the one or the other point of view: If we think of two masses coming into existence in space, suddenly, out of nothing and attracting each other immediately, then, evidently, this is all what happens – the simultaneous presence of masses. If, however, the mutual attraction starts off only some time after the masses have been brought into the space, then, we have to think of a specific condition which starts off propagating to the surrounding space, a condition which, in contrast to the foregoing state, could be called a state of tension (Zwangszustand) giving rise, as soon as it reaches the other mass, to a movement of that mass. In the first case, the space around the mass remains indifferent as long as the mass is left alone, and it will remain so also in the presence of another mass. The space would be only there as a separating distance. In the second case, the state of tension is present in the environment of a mass independently of the presence of other masses.

It has to be realized that the state of tension is not an empirical fact but follows simply from the concept of a successive propagation giving reason to this fact. If the propagation is presumed without having a forgoing idea about this state and about its arising consequences, then we will get embarrassed by contradictions. Similarly as the space around an electrically charged body is called an electric field which is to express that its properties have undergone an change by the charge, so it must be said: If a mass is brought into space, a gravitational field will ensue, i.e. a certain alteration will start off the circumference of the mass and propagate ever further to the periphery. The essence of this alteration remains unknown; we just take notion of its existence by the fact that another mass within the gravitational field is subject to attraction. The concept of alteration is even exhausted by this fact; It would be misleading to consider and explore this change as something separated from the masses. The only thing we can do is to look for the spatial and temporal relationships which, however, wouldn’t teach anything new as far as to the essence of the attraction. Such a relationship is the temporal dimension of the spatial progression of the tensed state, on which the speed of gravity rests.

If this should appear all too abstract, one might remember that nothing else can be affirmed, e.g., about the propagation of light. Well, it can be shown that a periodic process must be going on along the ray of light; nevertheless, nobody knows what it is which increases and decreases periodically, even it is named electric or magnetic force. Anyway, this name only denotes, apart form the visible and thinkable effect on bodies which have been brought to the appropriate place, some undefined state which, after all, has no need to be defined. The abstract notion maintaining that the spatial propagation of mass attraction resides in the propagation of a not further characterized state of spatial tension is not so void that the influence of the speed of gravity on the movement of masses would not be permitted being recognized. In restricting ourselves to the limits of our comprehension we are bewared from inferring more than what is adequate for the subject and the presumption.

Let us consider two masses of, for sake of simplicity, arbitrarily small size. To make a difference, one of them may be called attracting, the other one attracted, both being at rest in a given distance from each other. Now, the state of tension has time to propagate from one mass to the other and back again. Thus, this state of tension will be given in each of the different elements of space around the masses as well as along the distance between them and may be measured according to an appropriate dimension. Dividing the distance into infinitely small zones we may consider the size of the tensed state within each zone to be constant. Hence, the attracting zone of the attracting body at the distance *s* from the attracted body will coincide with the attracting zone 1 of the attracted body giving rise to a combined state of tension. Once relaxed from their positions, each mass will start moving toward each other following Newton's law. Accordingly, if the attracting mass is shifted by the width of t zones and if the attracted mass is shifted by the width of 1 zone, then, having kept both masses for some time at rest, it will follow that the attracting zone s – t – 1 coincides with the zone 1 of the attracted mass and, starting from this new coincidence, the movement will obey again Newtons's law. Both, after the first and the second step, the respective redistributed states of tension which issued from the masses at rest will propagate in space. If, however, the masses from their first position at rest pass on to the second position, then the attracted mass will fail to reach the zone s – t – 1 issued from the attracting mass, but will have reached only the zone s – 1. This is, because the redistributed states of tension issued from the attracting force have reached not yet the attracted mass implying that the original state of tension still persists. Nevertheless, it must be born in mind that the state of tension S – 1 issued from the attracting mass does not coincide with the state 1 issued from the attracted mass – which would have lead the state of tension nearby the attracted mass to adopt the distribution which, with the masses at rest, the state of tension would have had at a distance reduced by the width of 1 zone. Because, while the attracted mass is travelling the width of 1 zone, ever new arrangements of tensed states will be spreading from the attracted mass. Therefore, having reached the end of this zone, the state of the first zone will just be starting to form causing the new adjacent zone to be reached only partially by the states issued from former positions. This implies that part of this new zone will still be penetrated by the state of tension of the first zone, whereas another part will already be penetrated by the state of the following zone.

The last mentioned point may become clearer if, in addition, we consider the impossibility of the converse. Because, if in that instant when the attracted mass has traversed the width of one zone, the width of the now adjacent zone comprising the state of the zone S – 1 occupied by the attracting zone were, at the same time, occupied totally by the state of zone 1, then it would follow that, while the attracted mass is travelling an infinitely small distance of second order, the tensed state should have spread by an infinitely small distance of first order – i.e. it should exhibit an infinitely large velocity, which is against our proposition. Also, for the case of the attracting zone at rest – in which case the zone S - 1 and 1 were occupied by each other - , it would be all the same to assume either that the gravitic field is firmly attached to the attracted mass so that it would penetrate with the velocity exhibited by this mass into the gravitic field of the attracting mass or that the gravitic zones of the attracted mass penetrate, so as to say detached from the mass, with a velocity different from that mass, into the zones of the gravitic field of the attracting mass – thus complying with the proposition of a finite propagation velocity of the tensed state and creating a different type of superposition of both zones and, therefore, yielding a value of attraction which differs from the value obtained under the first assumption.

The motion of the attracted mass, therefore, not only does not obey Newton's law –as would be the case for the actual distance with the masses at rest - but, in addition, the deviation from this law could not be accounted for by the distance of the attracted mass from the site occupied by the attracting mass, when the tensed state reached were spreading thereof. Because, the attracted mass itself determines, by virtue of its velocity, together with the propagation velocity of the tensed state, its own behaviour in the given position. This situation is at variance from e.g. the propagation of light involving simply the transit of light from one body to another. On the contrary, the gravitational motion of the mass is dependent on the state of the space in its immediate environment, this state being influenced at the same time by the moving mass itself as well as by the presence of other masses.

Once one wishes to find out in more detail the alteration which Newton's law must undergo accordingly, it becomes evident that it is not all the same whether this law is applied in its form as accelerating force or as potential. It can be shown that it is impossible, by applying solely Newton's law and by assuming a finite propagation velocity of the tensed state, to find out the force by which the masses are accelerated. Instead, it is possible to determine only the respective potentials.

Beside of the presupposed propagation velocity of gravity, Newton's law is the only underlying fact from which the alteration in question could be deduced and its validity can, because of this alteration, strongly be demonstrated only for the resting state or for the case of a transition from rest into motion. Therefore, to reach the proposed goal, no other mean is feasible than to show that, given a certain distance between the masses in motion, the gravitic action exhibits the value at another distance than it would have had, according to this law, at the resting position. In other words, one needs find out those distances for which the tensed state nearby one or both masses will, for the case of motion, have the same distribution as for other certain distances at rest. The expression for the accelerating force fails in this case. Because, given as a directed quantity and being dependent on the distribution of the tensed state around the mass in motion, it will not be the same, whether a given distribution of the tensed state were in a state of equilibrium issued from the masses at rest or whether it were in a state of continuous change caused by the masses in motion. In the first case the tensed state will be endowed with a directed and directing activity of its own; in the second case, however, the directed and directing activities have to be taken in account which the respective mass is able to exert by its motion and which the tensed state is able to exert by its propagation. Exploring the relationships of each of these activities would require some notion on the qualities of the tensed state. This would exceed the concept derived from our assumption of a finite propagation velocity of gravity and is, therefore, out of the question.

The situation changes, if we address our question from the part of the potential. This potential is given, according to Newton, by the work to be done in order to move the attracted unit of mass from a given distance into infinity, provided the velocity by which this is done is considered not relevant allowing it to have an arbitrarily small value near zero. Its size in question during the movement equals the respective work, if the velocity assumes a recognizable constant value. The spatial or spatio-temporal change of the potential, multiplied by the quantity of the attracted mass, determines the increase or the decrease of the "living force" and of the potential gravitic energy between the masses during the transition from one position to another. Hence, it determines also the accelerating forces. The potential, therefore, must depend on the tensed state surrounding the mass. But, being no directed quantity, its value cannot be influenced by the directed and directing actions elicited by the tensed state itself by virtue of its propagation or by the motion of the mass. That is: if the distributions of the tensed state surrounding the mass are identical at rest and in motion, then this must hold also for the potentials elicited by the tensed state acting upon the mass.

The difference described entails, finally, that only the potential but not the accelerating force gives rise to the velocity. Indeed, in my previous paper I started right off with the potential. Properly speaking, neither "propagation of the potential" nor "propagation of the accelerating force" are adequate expressions. The real thing spreading successively is the tensed state. Since this state is linked to the given potential surrounding the mass, be it at rest or in motion together with the remaining masses, one is allowed to say that the potential arrives at the mass together with the tensed state. One may even say: it arrives at this location whether there is a mass or not; because the potential tempts to adopt always the value resulting from the tensed state as soon as the mass is placed there. The accelerating force, in contrast, being partially dependent on the co-action between the directed locomotion of the tensed state and the directed quantity of motion of the mass, it would make no sense to speak thereof without reference to the mass. Therefore, the predication of "spatial propagation", i.e. of a transition from one to another mass through space devoid of masses, lacks any underlying concept.

Having characterized the potential in that way, it remains to be seen, if the actual value of the potential acting upon the attracted mass can be determined. Because, so far, we had only an estimate up to which level the value of the surrounding tensed state could give rise to the potential, but not if it actually did so in any case. Imagine again the attracting and the attracted mass were both placed at once in the space. If their motion is thought to start off at once, then the potential must be immediately there. If one presupposes, however, that the motion is being initiated only some time thereafter, because the tensed state is propagating gradually, then, supposing that the arrival of the tensed state at the attracted mass and the incidence of the full value of the potential of the attracting mass occurred simultaneously, would imply an interruption of this continuity. In that case, even if the dimension of the attracted mass were infinitely small, the propagation of the activity of the tensed state, would take not only an infinitely small period of time but indeed no time at all. Hence, the action of the tensed state would spread without any time lag or within an infinitely small time interval of higher order over an infinitely small distance of first order and, therefore, would propagate with infinitely large velocity. Nevertheless, since the potential proceeds together with the mass and not in space devoid of mass, its development from zero to the full value has nothing in common with the process of the before mentioned propagation. The potential is transmitted to every part of the mass not at once but step by step. How much time this process takes, with the mass being at rest, is out of question. In any case, this time interval is proportional to the propagation velocity of the tensed state. Hence, if for some reason the propagation velocity increased while the Newtonian potential were constant, then the time in question for the transmittance should come out proportionally shorter; otherwise one would have to assume that the potential of the attracting mass acting upon the attracted mass would get transformed into the corresponding motion only some time after the arriving tensed state had passed the attracted mass. If the attracting mass manifests a velocity component in direction to the attracted mass, then the associated tensed state will spread proportionally faster to the attracted mass than it did with the attracting mass at rest. Because, if the tensed state penetrates by its own within the unit of time the width of one zone, then, necessarily, with the mass in motion it must advance further by the distance which the mass meanwhile progressed. If, furthermore, the attracted mass meets the tensed state half way with a certain velocity, then they will pass each other with the sum of both velocities. Hence, with both masses in motion, the time for transmitting the potential will be given by summing up the propagation velocity of the tensed state and the velocities of the masses. The potential formed by the tensed state would have not enough time to develop to the same extent as it were with the masses at rest. Accordingly, a smaller value of the potential must result.

Having exposed the entire sequence of ideas needed to prepare, in its original form of my previous treatise, the derivation of the potential exerted by two gravitating masses on each other during their motion, everything is set forth which needs be considered to valuate older attempts to calculate the propagation velocity of gravity. It will, thus, become evident that the concept of a propagation velocity and its influence on the gravitic attraction is nothing that can be judged solved already by applying the concept of velocity only. It is to be stressed explicitly that the notions unfolded here are but consequences of the original assumption that the gravitic activity needs time to spread in space. I have avoided introducing hypothetic elements into the sequence of my reasoning. Therefore, any further inferences are stipulated by this assumption only and any method of computation which does not conform to this stipulation must be considered insufficient.

III.

A detailed review concerning older attempts to calculate the propagation velocity of gravity has been published by O p p e n h e i m in 'Jahresberichte des K.K. akademischen Gymnasiums', Vienna, school year 1894-95. The title of the treatise is: 'Zur Frage nach der Fortpflanzungsgeschwindigkeit der Gravitation' (‚On the question concerning the propagation velocity of gravity'). This review was intended as a report only and, hence, refrains from criticizing the reported methods. The review is, however, very comprehensive because it reports not only on the foundation of the calculus but also on the calculus itself. Since we wouldn't refer to these, it will be enough, for this purpose, to refer to Oppenheim's treatise.

The merit for first having stimulated the question about the propagation velocity of gravity pertains to L a p l a c e. He dealt with this question in the year 1805 in Vol. IV of his "Traité de la Mécanique céleste", Chapter 7 of book 10. It was obvious to look for the traces of this velocity by investigating the orbits of moon and planets. The attention was therefore immediately drawn back to the way by which the propagation velocity of light in the universe has been assessed. Laplace understood, however, that there must be a difference between light and gravity with respect to the following important aspect: Light does not traverse every body and can be shielded as is the case, e.g., with the eclipse of Jupiter's moons. Gravity, on the contrary, is acting uninterruptedly once it has arrived at another celestial body and is not shielded by an intervening celestial body. Even today it seems not superfluous to remember this fact. In 1873, Z o e l l n e r proposed to determine the propagation velocity of gravity by measuring the temporal delay of Hengler's horizontal pendulum in indicating the sun's highest azimuth during the day. This proposal has been put forward again on occasions, although it could not be pursued, because, in fact, the expected delay must be absent for the mentioned reason. The velocity of light can, indeed, be determined if its propagation is not shielded by some body. This is evidenced by the method of aberration. The position of a fixed star appears to be shifted into the direction of the orbital movement of the earth by an amount given by the quotient of both velocities. Laplace argued that this should hold, likewise, for gravity. He imagined the attracting force hitting, say, a planet not along its communication line with the sun but in a somewhat tilted forward direction. Its orbital movement should, therefore, behave as if, apart from its acceleration toward the sun, there were an additional disturbing force acting in perpendicular direction to the radius vector. This force entered the calculus in direct proportion to the orbital velocity and in inverse proportion to the propagation velocity of gravity, while the attractive force itself remained unchanged obeying Newton's law.

The following has to be noted in this respect. Two masses left abandoned in total freedom to their gravitation and, thus, moving in direction to each other, would behave, according to Laplace, as if there were no finite propagation velocity – which is wrong following the former considerations. In addition, if the attraction which, according to Newton, is given at a certain distance between the planet and the sun were really acting at this given distance, but were modified only with respect to its direction, then this would point to a genuine force at distance, i.e. to a force arising simultaneously with the distance. By the way the concept of a gradual propagation is eliminated. Of course, these contradictions are not so much the point where Laplace was wrong. He imagined the gravitic field of the sun at rest while the planets accomplished their orbit in dependence of this field. The sun is, however, first of all not at rest, but is giving rise to a constantly renewed gravitic field, which is not shifted as a whole as it would be if a pure force at distance exhibiting an infinite propagation velocity were at work. Secondly, the sun influences the space traversed by the planet not by its own alone, but, instead, is influenced by the remaining variable gravitic field of the planet. From this even a particular difficulty arises for Laplace's model.

As soon as the influence of the propagation velocity of gravity is incorporated into a general law for the movement, encompassing Newton's law, it will turn out irrelevant whether perhaps, independently from the attractive force, the celestial bodies were subject to another type of movement, e.g. to a component of movement perpendicular to their orbit. As long as such a general law is not established beforehand, it is necessary to scrutinize anyway in which manner the directed activities of the tensed state elicited actually in the spatial environment by the moving mass itself as well as by other masses, are to be combined with the directed movement of the mass itself. Laplace committed the error, first, not to consider the activity elicited by the planet itself and, second, even more importantly, to have assumed that the attraction by the sun was fully expressed already before it encountered with the planet and as if the planet hadn't started its movement yet. These errors can be understood easily given the fact that Laplace and his comtemporaries were used to objectify the attraction at distance as if it were some seizable thing. Hence, it has been overlooked that such a conception was at variance with the assumption of a finite propagation velocity of gravity.

Around 40 years later, further elaboration of the theory of the 'activity at distance' lead, within the theory of electricity, to Weber's fundamental electrodynamic laws. The new constant appearing therein could, from the onset of the law, have had nothing to do with the gradual propagation of the electric attraction and repulsion. G a u ß, however, broached the question if Weber's or other one's fundamental law could be derived, provided such a gradual propagation existed. In that way, step by step, the constant was identified with the alluded value of propagation. In this respect the law of R i e m a n n of the year 1867, published in Poggendorff'sche Annalen, Vol. 131 and C. N e u m a n n's investigation on Weber's law, documented in his ‘Prinzipien der Elektrodynamik’ (1868) are particularly relevant. Thereby the idea came up to conceive Weber’s law as a generalization of Coulomb’s fundamental electric law, akin to Newton’s law of gravitation, and, hence, to conceive Weber’s law as a generalization of Newton’s law itself. With this in mind, one began to apply the fundamental electrodynamic laws for the orbital movements of the planets and to examine, whether the characteristic constant of these laws could be conceived as the propagation velocity of gravity. The incentive for this idea came from H o l z m u e l l e r 1870 in Vol. 15 of the ‘Zeitschrift fuer Mathematik und Physik’, followed by T i s s e r a n d 1872 in Vol. 75 of ‘Comptes Rendus’, by S e r v u s 1885 in his inaugural thesis and by L é v y 1890 in Vol. 110 of ‘Comptes Rendus’. Finally, in the treatise mentioned above, Oppenheim performed the computation according to the fundamental electrodynamic law of C l a u s i u s.

All these investigations failed to indicate the value of the eventually given limit of the propagation velocity of gravity or to find out if there were a limit at all. Three reasons may account for this. First, it has seemingly been overlooked that any coincidence found in this way between the spatial propagation of gravity and the electrodynamic activities should yield for the constant in question not the value equalling the velocity of light itself but this value times $\sqrt{2}$, since in Weber’s law the constant exhibits this value. For this reason alone, Lévy’s attempts are untenable, by combining arbitrarily and without any foundation, Weber’s and Riemann’s laws and using the value of the velocity of light, to assess mercury’s perihelion advance. Second, the discrepancies between the constant for the speed of light and Weber’s constant give no clue to establish whether this constant has indeed to be taken as the propagation velocity of gravity. Third, all computing attempts lack tackling the conceptual question, whether one or the other fundamental electrodynamic law could be applied to the concepts concerning the phenomenon of gravity and could, thus, at least by proxy be justified. In general, this application must be questioned, because the attraction between masses and the attraction between charges differ in their properties, e.g. by their duplicate attractive-repulsive nature, not to speak of the questionable validity of the relevant laws in their own domain.

Nevertheless, attempts to apply the fundamental electrodynamic laws to the sidereal revolutions within the solar system have not been totally vain. They have set in flux Laplace’s problem which for decades nobody was able to set about and helped recognize that the successive propagation of gravity, if it existed, should be reflected in the laws for the motion of masses, implying a modification of Newton’s law and being identical if the masses were at rest. Thanks to these attempts, the problem has been settled down as a matter of gravity itself and its solution has got a better, although not a sufficient foundation.

This solution has been set forth in 1885 by L e h m a n n – F i l h é s in Vol. 100 of the ‘Astronomische Nachrichten’ under the title: “On the movement of a planet by assuming a gradually propagating gravity”, - and 1888 by J. v o n H e p p e r g e r in Vol. 97 of the “Sitzungsberichte der K.K.Akademie in Wien” under the title: “On the propagation velocity of gravity”. Lehmann-Filhés final result is negative asserting that mercury’s perihelion advance could not be explained by the alluded assumption. Heppergers investigation was intended to establish a lower limit for the propagation velocity of gravity being compatible with the astronomical facts. He found this limit to be 500 times larger than the velocity of light. Both investigators start from the idea, that the solar incentive would meet the planet at a time when its distance from the sun differed from the distance at the time when the incentive was sent out. Hence, they determined the incentive according to Newton’s law while correcting for the distance of the planet from the location occupied by the sun at the time when the incentive left. Thus, the equations for the planetary motion retain their form following Newton’s law, but the coordinates of the sun are shifted according to the time needed by the gravitic action to be transmitted from the sun to the planet and according to the spatial forward velocity of the sun. Correspondingly, the planetary incentive meeting the sun is, of course, shifted in the same way so that the equations of the solar motion are adapted to the values of the planetary coordinates at the time when the incentive left the planet.

Thereby, a considerable progress beyond Laplace is achieved. Whereas the famous author of the celestial mechanic allowed only the direction, but not the quantity of the incentive meeting the planet to be influenced by the successive propagation of gravity, Lehmann-Filhés and Hepperger were aware that, both, the motion of the sun, together with the lag of time during the propagation of gravity, as well as the motion of the planet together with the respective lag of time altered, both, the attraction exerted on the sun and the attraction exerted on the planet as compared to the issue with both masses being at rest. There is no need, however, to demonstrate that the motion of the sun and of the planet are far from being already defined, although Lehmann-Filhés and Hepperger believed it were. Using the definitions given above, the situation is as follows: The tensed state s – 1 issued from the sun cannot coincide with the tensed state 1 issued from the planet, instead, the state s – 1 will have to combine partially with the tensed state 1 and partially with the newly arising state 2. Even if state s – 1 and state 1 coincided, it would at least be doubtful still, if they would elicit the same accelerating force in the moving planet as with the planet and the sun up to this point at rest and their motion just starting now. I wished to refer to the result of the considerations above, because indeed the concept advanced by Lehmann-Filhés and Hepperger can, by the way, be brought to an end. Let the differences between the rectangular coordinates of the sun in its actual position and of the planets at a time, when the activity now reaching the sun issued thereof, be x_1, y_1 and z_1. Let, further, the actual distance of the sun from the position occupied by the planet at that time by r_1. Hence, according to Lehmann-Filhés and Hepperger, the force components acting on the sun will be proportional to x_1/r_1^3, y_1/r_1^3 and z_1/r_1^3. Further, let the differences between the actual rectangular coordinates of the planet and those of the sun, when its action reaching the planet was issued thereof, be x_2, y_2, and z_2 and let the actual distance of the planet from the previous position of the sun be r_2. Accordingly, the force components acting on the planet will analogously be proportional to x_2/r_2^3, y_2/r_2^3 and z_2/r_2^3. It is easy to see that

$$x_1/r_1^3 = - x_2/r_2^3, \quad y_1/r_1^3 = - y_2/r_2^3 \text{ and } \quad z_1/r_1^3 = - z_2/r_2^3$$

cannot hold. If this were true one would get, squaring and adding all three equations and taking into account $x_1^2 + y_1^2 + z_1^2 = r_1^2$ and $x_2^2 + y_2^2 + z_2^2 = r_2^2$, thus $r_1 = r_2$, $x_1 = - x_2$, $y_1 = - y_2$ and $z_1 = - z_2$, yielding identical trajectories for the sun and the planet. Consequently, according the Lehmann-Filhés and Hepperger, the force components of the sun and the planet will differ from each other, precluding the activities to be balanced. This imbalance is, by no means, caused by some forces acting from outside onto the sun and the planets, since fractions x_1/r_1^3 etc. keep their significance, even if exterior forces were absent. A free mechanical system must be in equilibrium if its centre of gravity is either at rest or is moving linearly with constant velocity. If we were to ask, why the statement of Lehmann-Filhés and Hepperger is wrong in this respect, we will find the following answer: Since they stated correctly (1) the impulse acting on the planet is dependent on the position of the sun and (2) the impulse acting on the sun is dependent on the position of the planet, it follows that, with reference to (1), the motion of the planet and, with reference to (2), the motion of the sun must exert some influence. This inference entails considering the properties and alterations of the space surrounding the masses; because it could not be understood, how an indifferent environment could make a difference between impulses hitting the mass in faster or slower motion or at rest. These considerations gave rise to the sequence of judgements culminating in the first two main statements of section II.

The third main point dealt with in this chapter has neither by Laplace nor by Lehmann-Filhés and Hepperger been taken into account. The totally immediate application of the fundamental electrodynamic laws to the motion of the planets precludes such a consideration. We have seen, however, that apart from these, also elder attempts to prove and compute the propagation velocity of gravity have been unsuccessful. Mechanical theories of gravity by Hooke as exposed comprehensively in the year 1897 by Drude in his lecture "actions at distance" for the 69. Assembly of German Natural Scientists and Physicians, as well as the electric theory, exposed in the year 1900 by H.A.Lorentz in his lecture "reflections on gravity" for the Assembly of the K.Academy of Amsterdam either fail to deal with the propagation velocity of gravity or presuppose it to be identical with the velocity of light. These statements, therefore, are irrelevant for our discussion whether there is a finite propagation velocity of gravity. Conversely rather, the decision of this question could be relevant for them. Thus, from the standpoint of my previous treatise as well a from the present inquiry, it comes out that Lorentz's theory must either be wrong or be corrected, because, according to this theory, the time lag associated with the propagation of gravity, is unable to generate the advance of mercury's perihelion. Moreover, mechanical theories are totally unsuited to solve the problem of the propagation velocity of gravity. They propose mechanical models for non-mechanical processes of gravity. Whose inferences are contrafactual making it impossible to judge how far they would reach. If they were to stimulate decisive experiments and observations then they would prove their utility; the decisive result, however, were owed to the experiments and observations, not to the theory itself. Other theories, such as Lorentz's electrical theory, are of better quality. They infer a connection between different sorts of physical processes which might be factual indeed. Up to now, however, nothing of this type has been found as far as gravity as a whole is concerned. It will even be more difficult to find out, by this way, something about its velocity of propagation.

IV.

It is to be shown now, in which way it is possible, by applying the main points established in section II, to determine the influence of the gradual propagation of gravity on the movement of masses, to derive the potential mentioned in my previous treatise and, finally, to define the modification of Newton's law which follows thereof.. To complete my task, the procedure of computation for Mercury will basically be added as an example.

Let Mass m be located in point A and mass m' in point B, respectively, both being of arbitrary small extension. Their distance be $r - \Delta r$, Δr being negative, because the attraction decreases with r. The masses be at rest initially, so that the potential of m upon m' will be, with $\mu' = Gm$,

$$\mu' / (r - \Delta r)$$

Let mass m' be fixed in point B, while mass m is set free. Hence, m will move in time dt in the direction of m' travelling the distance $AC = -dr$. We know: the gravitational field of m' remains unchanged, but starting from m a new arrangement of tensed states is spreading over the surrounding space. Therefore, the tensed state in the surroundings of m will be different when m is arriving in C as it were if this mass remained in point A at rest. It is impossible, therefore, to assess the potential of the tensed state generated by m. The tensed state issued from m, when reaching C, will spread further arriving in time $\Delta t - dt$ at m' , where the same state is created as if m remained at rest in C. Hence, the potential of m acting on m', - linked, anyway, to the actual ability of the tensed state – must equal the potential for the distance $r - \Delta r + dr$, if m were at rest, i.e. it will be equal

$$\mu' / (r - \Delta r + dr)$$

The corresponding distance between *m* and *m'* is now, *m* having travelled from *A* in time Δt the distance $AD = -\Delta r$, different equalling *r*. Moreover, the tensed state will propagate, unlike if *m* were at rest and, hence, the original propagating velocity were *c*, with the additional velocity $-dr/dt$ which *m* exhibits in *C*. The total velocity of the tensed state when passing *m'* will, therefore, be $c - dr/dt$. The time needed to communicate the potential to *m'* is shortened in this way by the relation of *c* to $c- dr/dt$. Hence, the actual potential acting upon *m'* and inciting, by the way, *m'* to start its movement, if *m'* were relaxed in this moment, will be:

$$\frac{\mu' c}{(r - \Delta r + dr)\left(c - \frac{dr}{dt}\right)}$$

The differential thereof, multiplied by *m'*, yields the infinitely small increase of the 'living force' of the masses in the following element of time. It is possible, therefore, by forming the inner product of the potential and the mass *m'* or by applying Lagrange's equations of motion to the 'living force', to derive the accelerating force between *m* and *m'*. The potential of *m'* acting upon *m* is, therefore, equal

$$\frac{\mu c}{(r - \Delta r + dr)\left(c - \frac{dr}{dt}\right)}$$

With $\mu = \mu' m'/m$. Otherwise the accelerating force acting from *m'* upon *m* would be obtained as differing from the force acting conversely from *m* upon *m'*. This cannot be because the forces acting against each other need be balanced for the formerly given reason.

Now we assume *m* being fixed in *A* and *m'* is moving. Hence, we will obtain, applying the same considerations, the same values for the potentials, except for the distances *AC* and *AD* equal $-dr$ and $-\Delta r$, respectively, which are to be substituted by the distances *BC'* and *BD'*. Of course, it is irrelevant how long mass *m* rested in *A* or mass *m'* rested in *B* before each of them is released. Therefore, the same sequence of inferences may immediately be applied to the positions of the masses in *D* and *B* as they were in *A* and *D'* and may be performed in the same manner. Hence, the expressions for the potentials are valid for every distance.

The sole question remains what comes out if both masses are released at once. The same distribution of the tensed state elicited ìn *D'* by solely moving *m'* will be given, but *D'* will be located nearer to *B*, because the tensed state issued by *m* will arrive earlier by the quantity of speed of this mass. The potential of *m* acting upon *m'* will, therefore, be determined by the same expression as for the masses at rest, thus, as if *m* were fixed. Hence, Newton's law may be applied giving $CC' = AB - AC - BC'$. In addition, the influence of the velocity of the tensed state while passing *m'* has to be taken into account. This velocity is composed from three velocity components, i.e. the propagation velocity of gravity, the velocity of the mass *m* and the velocity of *m'*. *AB* may be represented, as formerly stated, by $r - \Delta r$ if $AD + BD' = -\Delta r$; further $AC + BC'$ equals $-dr$; finally, the sum of the velocities of *m* and *m'* is $-dr/dt$. Therefore, the potential of *m* acting upon *m'* will again be given by

$$\frac{\mu' c}{(r - \Delta r + dr)\left(c - \frac{dr}{dt}\right)}$$

There is no need to demonstrate that, with both masses in motion, for the potential of *m'* acting upon *m*, it must likewise follow:

$$\frac{\mu c}{(r - \Delta r + dr)\left(c - \frac{dr}{dt}\right)}$$

This potential is named V. Dividing both the numerator and the denominator by c and lifting r we get:
(5)

$$V = \frac{\mu}{r\left(1 - \frac{\Delta r - dr}{r}\right)\left(1 - \frac{1}{c}\frac{dr}{dt}\right)}$$

Given the masses travel the distance $-\Delta r + dr$ while the tensed state spreads over the distance $r - \Delta r + dr$, it follows that the distances are proportional to the corresponding velocities, thus:

$$\frac{\Delta r - dr}{r - \Delta r + dr} = \frac{1}{c}\frac{dr}{dt}$$

where the distance $\Delta r - dr$ is vanishing compared to r. Otherwise Newton's law would not hold for masses in motion as it does. Hence, neglecting the corresponding expression, we get:
(7)

$$V = \frac{\mu}{r\left(1 - \frac{1}{c}\frac{dr}{dt}\right)^2}$$

Developing (7) up to the second potency yields:
(8)

$$V = \frac{\mu}{r}\left[1 + \frac{2}{c}\frac{dr}{dt} + \frac{3}{c^2}\left(\frac{dr}{dt}\right)^2\right]$$

As mentioned in section II, this formula expresses the work to be done, if the unit of mass m were brought with the velocity dr/dt to infinity. Its change during the motion of the mass in time dt over the distance dr yields, multiplied with m, the increase of the 'living force' T. Therefore, setting $dr/dt = r'$ and applying the Lagrangian equations of motion, we obtain for the acceleration of m
(9)

$$\frac{1}{m}\frac{dT}{dr} - \frac{1}{m}\frac{d}{dt}\frac{dT}{dr'} = \frac{dV}{dr} - \frac{d}{dt}\frac{dV}{dr'} = -\frac{\mu}{r^2}\left[1 - \frac{3}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{6r}{c^2}\frac{d^2r}{dt^2}\right]$$

In order to compute mercury's perihelion progression, the extension of the sun and the planet may be neglected. Moreover, it may be convenient to place the sun into the origin of the coordinates. In this case μ is augmented by the relation of the sum of the solar and the planetary mass to the mass of the sun. Setting:
(10)

$$\frac{3}{c^2}\left(\frac{dr}{dt}\right)^2 - \frac{6r}{c^2}\frac{d^2r}{dt^2} = F$$

The equations for the motion of the planet read:
(11)

$$\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3}(1 - F),$$

$$\frac{d^2y}{dt^2} = -\frac{\mu y}{r^3}(1 - F).$$

Multiplying the first equation with y and the second with x and subtracting we obtain:
(12)

$$x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = 0\,.$$

It is known from the derivation of the planetary movement according to Newton's law that, defining θ as the angle between the radial vector and the positive abscissa and L as a constant, it follows:
(13)

$$r^2 \frac{d\vartheta}{dt} = L$$

Making use of the following identities
(14)

$$dt = \frac{r^2 d\vartheta}{L},$$

$$\frac{x}{r} = \cos\vartheta,$$

$$\frac{y}{r} = \sin\vartheta,$$

The equations of motion read:
(15)

$$d\frac{dx}{dt} = -\frac{\mu}{L}(1 - F)\cos\vartheta \, d\vartheta,$$

$$d\frac{dy}{dt} = -\frac{\mu}{L}(1 - F)\sin\vartheta \, d\vartheta.$$

Integration yields with the constants M and N
(16)

$$\frac{dx}{dt} = -\frac{\mu}{L}\sin\vartheta + \left(M + \int \frac{\mu}{L} F\cos\vartheta \, d\vartheta\right),$$

$$\frac{dy}{dt} = \frac{\mu}{L}\cos\vartheta + \left(N + \int \frac{\mu}{L} F\sin\vartheta \, d\vartheta\right).$$

Therefore we get:
(17)

$$r = \frac{L}{\frac{\mu}{L} - \left(M + \int \frac{\mu}{L} F\cos\vartheta \, d\vartheta\right)\sin\vartheta + \left(N + \int \frac{\mu}{L} F\sin\vartheta \, d\vartheta\right)\cos\vartheta}.$$

This is the equation of an ellipse. Indicating the larger half-axis by *a* and the smaller half-axis by *b*, the numerical excentricity by *ε* and the angle between *a* and the positive abscissa by *ω*, one finds, if the three equations for $r = a(1 - \varepsilon)$, $r = a(1 + \varepsilon)$ and $r = b^2/a$ are formed:
(18)

$$L = b\sqrt{\frac{\mu}{a}},$$

$$M + \int \frac{\mu}{L} F\cos\vartheta \, d\vartheta = -\frac{\varepsilon}{b}\sqrt{a\mu}\sin\omega,$$

$$N + \int \frac{\mu}{L} F\sin\vartheta \, d\vartheta = \frac{\varepsilon}{b}\sqrt{a\mu}\cos\omega.$$

The last two equations are to be differentiated for *θ,* taking into account that $b/\sqrt{a}$ is not variable; Inserting the value of *L* and dividing the first equation by $\frac{\sqrt{a\mu}}{b}\cos\vartheta$, the other one by $\frac{\sqrt{a\mu}}{b}\sin\vartheta$ one obtains: (21)

$$F = -\frac{\sin\omega}{\cos\vartheta}\frac{d\varepsilon}{dt}\frac{dt}{d\vartheta} - \varepsilon\frac{\cos\omega}{\cos\vartheta}\frac{d\omega}{dt}\frac{dt}{d\vartheta},$$

$$F = \frac{\cos\omega}{\sin\vartheta}\frac{d\varepsilon}{dt}\frac{dt}{d\vartheta} - \varepsilon\frac{\sin\omega}{\sin\vartheta}\frac{d\omega}{dt}\frac{dt}{d\vartheta}.$$

Setting both expressions equal and introducing $a = \theta - \omega$ it follows: (22)

$$\frac{d\varepsilon}{dt} = -\varepsilon \operatorname{tang} \alpha \frac{d\omega}{dt}$$

And hence: (23)

$$F = -\frac{\varepsilon}{\cos\alpha} \frac{dt}{d\vartheta} \frac{d\omega}{dt}$$

It is possible to develop another expression for F according to its original meaning. Using the formulae

$$\frac{d\varepsilon}{dt} = -\varepsilon \operatorname{tang} \alpha \frac{d\omega}{dt},$$

$$r^2 \frac{d\vartheta}{dt} = L,$$

$$L = b\frac{\sqrt{\mu}}{\sqrt{a}}$$

one finds

$$r = \frac{\frac{b^2}{a}}{1 + \varepsilon\cos\alpha},$$

$$\begin{aligned}
\frac{dr}{dt} &= -\frac{a r^2}{b^2}\left(\cos\alpha \frac{d\varepsilon}{dt} - \varepsilon\sin\alpha\frac{d\vartheta}{dt} + \varepsilon\sin\alpha\frac{d\omega}{dt}\right) \\
&= -\frac{a r^2}{b^2}\left(-\varepsilon\cos\alpha \operatorname{tang}\alpha\frac{d\omega}{dt} - \varepsilon\sin\alpha\frac{d\vartheta}{dt} + \varepsilon\sin\alpha\frac{d\omega}{dt}\right) \\
&= \frac{a\varepsilon r^2}{b^2}\sin\alpha\frac{d\vartheta}{dt} \\
&= \frac{\varepsilon\sqrt{a\mu}}{b}\sin\alpha,
\end{aligned}$$

$$\begin{aligned}
\frac{d^2 r}{dt^2} &= \frac{\sqrt{a\mu}}{b}\sin\alpha\frac{d\varepsilon}{dt} + \frac{\varepsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\vartheta}{dt} - \frac{\varepsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\omega}{dt} \\
&= -\frac{\varepsilon\sqrt{a\mu}}{b}\sin\alpha\operatorname{tang}\alpha\frac{d\omega}{dt} + \frac{\varepsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\vartheta}{dt} \\
&\qquad - \frac{\varepsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\omega}{dt} \\
&= -\frac{\varepsilon\sqrt{a\mu}}{b}\sin\alpha\operatorname{tang}\alpha\frac{d\omega}{dt} + \frac{\varepsilon\mu}{r^2}\cos\alpha - \frac{\varepsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\omega}{dt} \\
&= -\frac{\varepsilon\sqrt{a\mu}}{b\cos a}\frac{d\omega}{dt} + \frac{\varepsilon\mu}{r^2}\cos\alpha.
\end{aligned}$$

Consequently, (24)

$$F = \frac{3\varepsilon^2 a\mu}{b^2 c^2}\sin^2\alpha + \frac{6\varepsilon r\sqrt{a\mu}}{b c^2\cos\alpha}\frac{d\omega}{dt} - \frac{6\varepsilon\mu}{r}\cos\alpha.$$

Equalizing the actual and the former expressions for **F**, and substituting *dt/dθ* by $\frac{r^2\sqrt{a}}{b\sqrt{\mu}}$, the equation for *dω/dt* reads: (25)

$$\frac{\varepsilon r^2\sqrt{a}}{b\sqrt{\mu}\cos\alpha}\,\frac{d\omega}{dt} = -\frac{3\varepsilon^2 a\mu}{b^2c^2}\sin^2\alpha - \frac{6\varepsilon r\sqrt{a\mu}}{bc^2\cos\alpha}\,\frac{d\omega}{dt} + \frac{6\varepsilon\mu}{r}\cos\alpha\,,$$

Introducing:

$$r = \frac{b^2}{a(1+\varepsilon\cos\alpha)} \quad (26) \quad \text{and} \quad b = a\sqrt{1-\varepsilon^2} \quad (27)$$

and dividing by
(28)

$$\frac{\varepsilon r^2\sqrt{a}}{b\sqrt{\mu}\cos\alpha}$$

yields: (29)

$$\begin{aligned}\frac{d\omega}{dt} = &-\frac{6\mu}{a(1-\varepsilon^2)c^2}(1+\varepsilon\cos\alpha)\frac{d\omega}{dt}\\ &-\frac{3\varepsilon\mu^{3/2}}{a^{5/2}(1-\varepsilon^2)^{5/2}c^2}(1+\varepsilon\cos\alpha)^2\sin^2\alpha\cos\alpha\\ &+\frac{6\mu^{3/2}}{a^{5/2}(1-\varepsilon^2)^{5/2}c^2}(1+\varepsilon\cos\alpha)^3\cos^2\alpha\,.\end{aligned}$$

This equation is multiplied by *dt* and the following expressions are inserted in the second and third member at the right side: (30)

$$dt = \frac{r^2}{L}\,d\vartheta = \frac{a^{3/2}(1-\varepsilon^2)^{3/2}}{\mu^{1/2}(1+\varepsilon\cos\alpha)^2}(d\alpha + d\omega)\,.$$

Rearranging conveniently and dividing yields: (31)

$$= \frac{\frac{6\mu}{a(1-\varepsilon^2)c^2}(1+\varepsilon\cos\alpha)\cos^3\alpha - \frac{3\varepsilon\mu}{a(1-\varepsilon^2)c^2}\sin^2\alpha\cos\alpha}{1+\frac{6\mu}{a(1-\varepsilon^2)c^2}(1+\varepsilon\cos\alpha)-\frac{6\mu}{a(1-\varepsilon^2)c^2}(1+\varepsilon\cos\alpha)\cos^3\alpha+\frac{3\varepsilon\mu}{a(1-\varepsilon^2)c^2}\sin^2\alpha\cos\alpha}\,d\alpha\,.$$

Dividing the numerator and denominator by: (32)

$$\frac{3\mu}{a(1-\varepsilon^2)c^2} = \frac{\gamma}{c^2}\,,$$

and arrangement following increasing potencies of *cos α*, with:

$$\begin{aligned}-\varepsilon\cos\alpha + 2\cos^2\alpha + 3\varepsilon\cos^3\alpha &= v\,,\\ 3\varepsilon\cos\alpha - 2\cos^2\alpha - 3\varepsilon\cos^3\alpha &= w\,,\end{aligned}$$

Yields: (33)

$$d\omega = \frac{v}{\frac{c^2}{\gamma}+2+w}\,d\alpha = \left[\frac{v}{\frac{c^2}{\gamma}+2} - \frac{vw}{\left(\frac{c^2}{\gamma}+2\right)^2}\right]d\alpha\,.$$

Thus, the perihelions advance ψ during one sidereal revolution of the planet comes out: (34)

$$\psi = \int_0^{2\pi} \left[\frac{v}{\frac{c^2}{\gamma} + 2} - \frac{v w}{\left(\frac{c^2}{\gamma} + 2\right)^2} \right] d\alpha = \frac{2\pi}{\frac{c^2}{\gamma} + 2} + \frac{3\pi(8 - \varepsilon^2)}{8\left(\frac{c^2}{\gamma} + 2\right)^2}$$

Therefore: (35)

$$\frac{c^2}{\gamma} + 2 = \frac{\pi}{\psi} + \sqrt{\frac{\pi^2}{\psi^2} + \frac{3\pi(8 - \varepsilon^2)}{8\psi}} .$$

Because ψ is small, the second member under the root vanishes away. It remains:

$$\frac{c^2}{\gamma} + 2 = \frac{2\pi}{\psi},$$

$$c^2 = \frac{2\pi\gamma}{\psi} - 2\gamma .$$

Again, 2γ can be neglected with respect to $2\pi\gamma/\psi$, so that one obtains: (36)

$$c^2 = \frac{6\pi\mu}{a(1 - \varepsilon^2)\psi} .$$

Substituting (37)

$$\mu = \frac{4\pi^2 a^3}{\tau^2},$$

wherein τ indicates the sidereal revolution time of the planet, and inserting the known values:

$a = 0.3871.149.10^6$ km
$\varepsilon = 0.2056$
$\tau = 88$ days
$\psi = 4.789.10^{-7}$

it comes out :

$c = 305.500$ km/sec

V.

The fact that physical processes need time for their propagation is one of the fundamental physical manifestations. The presence or absence as well as the quantity of the time lag involved may give a clue, whether processes can be conceived as ‘actions at near’ or as ‘actions at distance’ and which might be the medium wherein the processes take place. Therefore, the endeavour is always to find out, for any kind of processes and on several instances, the actual consume of time; moreover, these instances will be selected for the most simple and transparent cases. With respect to both properties, uncovering e.g. the propagation velocity of light will be likewise easier than revealing the propagation velocity of electrical waves. As far as gravity is concerned, my present and former attempts, of course, leave something to be desired. At least, the question about the propagation velocity of gravity will not be settled without employing the herein apprised modification of Newton’s law. It is possible, however, that there might be circumstances in which the general law can, eventually, be derived more easily and more concisely. The notion of the general law may help anyway finding such specific cases.

I wished to direct the attention of the reader to the following: The derivation of the general expressions for the potential and for the accelerating force of gravity are valid as long as the speed of the masses is low compared to the speed of gravity itself. And they will be valid also, if a sufficiently small speed is modified by external means, provided the centre of gravity of the attracting mass is mostly conserved avoiding uneven movements. This condition will be satisfied, if some body is uniformly brought down to the earth or if it is likewise raised. In these cases specific conclusions can be draw from the application of the general law which might lead to appropriate experiments and observations. Some doubts remain, however, whether, given the considerable value of the propagation velocity of gravity, the expected effects were not too small to be detectable.

If, else, our hopes proving the propagation velocity of gravity by other means should fail, the present demonstration, at least, will lend support to the notion that the medium sustaining the attraction of masses is identical with the medium propagating light, thermal radiation as well as electric and magnetic influences. Based on this notion one may construct safely a theory satisfying the ‘action at near’ of gravity. Of course, I have in mind a theory which gives not simply a mechanical explanation of the phenomenon but, instead, is able to establish the actual relationships between gravity and other physical phenomena.

Stargard i. Pommern, 16 February 1902.

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