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Deriving gravitation from electromagnetism

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We present a generalized Weber force law for electromagnetism including terms of fourth and higher orders in v/c. We show that these extra terms yield an attractive force between two neutral dipoles in which the negative charges oscillate around the positions of equilibrium. This attractive force can be interpreted as the usual Newtonian gravitational force as it is of the correct order of magnitude, is along the line joining the dipoles, follows Newton's action and reaction law, and falls off as the inverse square of the distance.

Nous présentons une généralisation de la loi de Weber pour la force électromagnétique, en incluant des termes d'ordre quatre et des ordres supérieurs en v/c. Nous montrons que ces termes supplémentaires donnent une force d'attraction entre deux dipôles neutres dans lesquels les charges négatives oscillent autour de leurs positions d'équilibre. Cette force d'attraction peut être interprétée comme la force gravitationnelle usuelle de Newton, car elle est du bon ordre de grandeur, est dirigée suivant la ligne qui joint les deux dipôles, obéit à la loi newtonnienne d'égalité de l'action et de la réaction et diminue comme $1/r^2$.

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1. Introduction

One of the main goals of physics is the unification of all the forces of nature in a single framework. Beyond aesthetical and theoretical reasons for this search, it is also expected that if it is achieved it can lead to important practical applications. As an analogy in the history of science, the theoretical, experimental, and technological developments in the nineteenth century following Oersted's discovery of the interconnection between an electric current and a magnet were enormous and many of them of a quite unexpected character.

A particular target of this general goal is the unification of gravitation with electromagnetism. Attempts in this direction have been many, with varying degrees of success. Faraday, for instance, devised experiments in 1850 to find a possible relation between gravity and electricity (1). He stated his motivation as follows:

> The long and constant persuasion that all the forces of nature are mutually dependent, having one common origin, or rather being different manifestations of one fundamental power, has made me often think upon the possibility of establishing, by experiment, a connexion between gravity and electricity, and so introducing the former into the group, the chains of which, including also magnetism, chemical force and heat, binds so many and such varied exhibition of force together by common relations.

Although the experimental effects he was looking for could be extremely small, he realized the importance of his search by saying:

> Such results, if possible, could only be exceedingly small, but, *if possible*, i.e. if true, no terms could exaggerate the value of the relation they would establish.

In his own words, his guiding idea was the following:

> The thought on which the experiments were founded was, that, as two bodies moved towards each other by the force of gravity, currents of electricity might be developed either in them or in the surrounding matter in one direction; and that as they were by extra force moved from each other against the power of gravitation, the opposite currents might be produced.

According to this view he let cylinders of copper, bismuth, iron, glass, etc. fall to the ground under the gravitational force of the Earth. He surrounded these bodies by a metallic helix connected to a galvanometer where he expected to find a signal during the fall of the cylinders. The helix was either fixed to the bodies so as to fall with them, or was kept stationary in the laboratory while the cylinders passed through it as they fell. In the end and after many variations of this experiment he could not find any effect. But nothing speaks better of the man Faraday, of his long held beliefs, and of the driving energy that was responsible for his discovery of electromagnetic induction 20 years earlier than his closing remarks at the end of these experiments (1) (our italics):

> Here end my trials for the present. *The results are negative. They do not shake my strong feelings of the existence of a relation between gravity and electricity*, though they give no proof that such a relation exists.

The scope of this paper is an initial exploratory endeavour along this general idea of trying to unify the gravitational and electromagnetic forces. As is well known, there is a strong relation between these two basic interactions. First of all there is the structural analogy between Newton's law of gravitation and Coulomb's force (both fall off as $1/r^2$; both are proportional to the product of a property of the interacting particles: the gravitational masses or the electrical charges; and both forces follow Newton's action and reaction law and are along the line joining the particles). Ever since the works of Meyer, Joule, and Helmholtz in the 1840's on the transformation and conservation of energy this interconnection has become much more evident. For instance, in modern hydroelectric power stations we transform potential gravitational energy into electromagnetic energy, while the opposite happens with any electromagnetic device intended to raise weights. Another line of reasoning indicating this mutual relationship is the fact that heavy neutral bodies such as an atom or a neutron have been broken into smaller charged particles such as protons and electrons. These facts

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suggest that all heavy bodies are composed of oppositely charged particles.

If we wish to unify gravitation and electromagnetism we will need to *derive* an equivalent to Newton's law of universal gravitation. Moreover we will need to explain why the gravitational force is approximately 10^{-36} times smaller than the electrostatic force at the same distance (here we are comparing the gravitational force between two hydrogen atoms or two neutrons with the Coulombian force between an electron and a proton). Whittaker has claimed that the first model trying to implement these conditions was due to the German physicist Aepinus in 1759 (2). According to Whittaker, Aepinus suggested that gravity might be a residual force arising from a slight lack of equality between electrostatic attraction and repulsion. As a matter of fact Aepinus never made such a suggestion, as has been discussed by R. W. Home in his introductory monograph to the first English translation of Aepinus' important work (3). The first to have made such a suggestion seems to have been Thomas Young in 1807 (3), and then it was advanced by Mossotti in 1836 (2, 4). Their idea was to suppose that the electric attractive force between unlike charges is slightly larger than the electric repulsive force between like charges of the same absolute magnitude. Nature behaving like this, there would remain a resultant attractive force between neutral atoms, which would be what we call gravitation. Wilhelm Weber (1875) and Friedrich Zöllner (1882) also worked with an idea of this kind (2, 5–8). To our knowledge none of them explained how the force between the charges could behave in this way, or what would be the source or origin of this slight imbalance in the electric forces. In this work we try to implement a variation of Young and Mossotti's ideas, presenting a constructive and quantitative model in which we derive these properties and the correct orders of magnitude.

Besides the general arguments above and the ingenuous insight of Young and Mossotti, there is another kind of reasoning leading to our idea. The fact is that magnetism is a second-order effect when compared with electrostatics, being essentially of the order v^2/c^2, where v is the typical velocity of the interacting charges and c is the velocity of light. This can be seen straight away in Lorentz's force law, according to which, the force on a charge q_1 is given by

$$\boldsymbol{F} = q_1\boldsymbol{E}_2 + q_1\boldsymbol{v}_1 \times \boldsymbol{B}_2$$

As a matter of fact the magnetic field is proportional to the electric current in the source, which, in turn, is proportional to the velocity of the source charges,

$$\boldsymbol{B}_2 \simeq \boldsymbol{v}_2 \times \frac{\boldsymbol{E}_2}{c^2}$$

This can also be seen by observing that the force between two current elements $I_1\, dl_1$ and $I_2\, dl_2$ has the order of magnitude given by (apart from geometric factors of the order of unity)

$$d^2 F \simeq \frac{(\mu_0 I_1 I_2\, dl_1\, dl_2)}{(4\pi r^2)}$$

where μ_0 is the vacuum permeability ($\mu_0 = 4\pi \times 10^{-7}$ kg m C^{-2}). Remembering that $c^2 = (\mu_0\varepsilon_0)^{-1}$, where ε_0 is the vacuum permissivity ($\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}) and that an electric current is usually understood as charges in motion ($I\, dl \rightarrow qv$), we find that the force between current elements is essentially weaker than Coulomb's force by a factor of v^2/c^2 (not considering, of course, the much larger number of charges involved in the interaction between electric currents). And the important fact is that typically, in ordinary situations, we have $v^2/c^2 \simeq 10^{-20}$ (see refs. 9 and 10). It is then natural to suppose that gravitation might be a fourth-order electromagnetic effect as this would give rise to the correct orders of magnitude.

In the following we present our model in which we try to explore these ideas coherently in a quantitative form.

2. Weber's law

As we want to derive gravitational effects we must begin with only electromagnetic forces. Our model of interaction will be based on a generalization of Weber's law. So, before we present our model, let us briefly review Weber's work and the main reasons why it is being developed and extended so vigorously nowadays. Following Oersted's discovery of the interconnection between magnetism and electric currents in 1820, Ampère began a long series of classical experiments to find an expression for the force between two current elements $I_1\, dl_1$ and $I_2\, dl_2$. In 1823 he obtained his final expression, which can be found in his work of 1825 (11). This work summarizes his main research in electromagnetism. Whittaker said that it is "one of the most celebrated memoirs in the history of natural philosophy" (see ref. 2, vol. 1, p. 83). In modern vectorial notation and utilizing the International Systems of Units the force exerted by $I_2\, dl_2$ on $I_1\, dl_1$ can be written, using Ampère's force law, as

$$[1] \quad d^2\boldsymbol{F}_{21} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{\boldsymbol{r}}_{12}}{r_{12}^2}[2(d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2) - 3(\hat{\boldsymbol{r}}_{12}\cdot d\boldsymbol{l}_1)(\hat{\boldsymbol{r}}_{12}\cdot d\boldsymbol{l}_2)]$$

In this expression $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the locations of the infinitesimal current elements $I_1\, dl_1$ and $I_2\, dl_2$, the distance between them is given by

$$r_{12} \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2| \equiv \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$\boldsymbol{r}_{12} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2, \qquad \hat{\boldsymbol{r}}_{12} \equiv \frac{\boldsymbol{r}_{12}}{r_{12}}$$

$\hat{\mathrm{r}}_{12}$ is a unit vector pointing from $I_2\, dl_2$ to $I_1\, dl_1$. It should be emphasized that Ampère's force complies with Newton's third law (action and reaction) in the strong form, namely, the force of $I_2\, dl_2$ on $I_1\, dl_1$ is not only equal and opposite to the force of $I_1\, dl_1$ on $I_2\, dl_2$, but it is also along the line joining them.

To unify electrostatics with magnetism so that it was possible to derive Ampère's force from a generalization of Coulomb's force, Weber proposed in 1846 and 1848 that the force exerted by the electric charge q_2 on q_1 should be given by (12, 13):

$$[2] \quad \boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{r}}_{12}}{r_{12}^2}\left(1 - \frac{\dot{r}_{12}^2}{2c^2} + \frac{r_{12}\ddot{r}_{12}}{c^2}\right)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{r}}_{12}}{r_{12}^2}\left[1 + \frac{1}{c^2}\left(\boldsymbol{v}_{12}\cdot\boldsymbol{v}_{12} - \frac{3}{2}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{v}_{12})^2 + \boldsymbol{r}_{12}\cdot\boldsymbol{a}_{12}\right)\right]$$

In this expression

$$\boldsymbol{v}_{12} \equiv \frac{d\boldsymbol{r}_{12}}{dt}, \qquad \boldsymbol{a}_{12} \equiv \frac{d^2\boldsymbol{r}_{12}}{dt^2} = \frac{d\boldsymbol{v}_{12}}{dt}$$

$$\dot{r}_{12} \equiv \frac{dr_{12}}{dt} = \hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{v}_{12}$$

$$\ddot{r}_{12} \equiv \frac{d^2 r_{12}}{dt^2} = \frac{d\dot{r}_{12}}{dt} = \frac{[\boldsymbol{v}_{12}\cdot\boldsymbol{v}_{12} - (\hat{r}_{12}\cdot\boldsymbol{v}_{12})^2 + \boldsymbol{r}_{12}\cdot\boldsymbol{a}_{12}]}{r_{12}}$$

Moreover, c is the ratio between electromagnetic and electrostatic units of charge, which was first determined experimentally by Weber and Kohlrausch in 1856 (14). They found its value, quite surprisingly at the time, to be the same as the known value of the velocity of light in a vacuum. This was the first indication of a strong interconnection between electromagnetism and optics (15, 16). In 1848 (13), Weber presented for the first time his generalized potential energy given by

$$[3] \quad U = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r_{12}}\left(1 - \frac{\dot{r}_{12}^2}{2c^2}\right)$$

His force law [2] can be derived from U by assuming that $\boldsymbol{F}_{21}$ is given either by the expression

$$\boldsymbol{F}_{21} \equiv -\hat{r}_{12}\frac{dU}{dr_{12}} \qquad \text{or by} \qquad \boldsymbol{v}_{12}\cdot\boldsymbol{F}_{21} = \frac{-dU}{dt}$$

so that it will comply with the virtual work concept.

Let us now discuss the main properties of Weber's law. Beyond his main works the better presentation of Weber's ideas can be found in the last chapter of J. C. Maxwell's *A Treatise on Electricity and Magnetism* (17), and in A. O'Rahilly's classical book (18). The first aspect to note is that Weber's force always follows Newton's action and reaction law in the strong form, which means that it is compatible with the conservation of linear and angular momentum. As his force law can be derived from a velocity-dependent generalized energy it also follows the principle of conservation of energy, although Weber himself only succeeded in proving this fact in 1869–1871 (19). So all the conservation laws of classical physics remain valid in Weber's theory. But it has another very important property, namely, it only depends on the relative distance (r_{12}), velocity ($\dot{r}_{12}$), and acceleration ($\ddot{r}_{12}$) between the interacting charges, which means that it always has the same value for all observers, irrespective of the states of motion, of q_1, q_2, and of the frame of reference. This is what we call a relational law, because it only depends on the relations between the interacting bodies.

When there is no motion between the interacting charges ($\dot{r}_{12} = 0$ and $\ddot{r}_{12} = 0$) we recover Coulomb's force from Weber's force. This means that all electrostatics as expressed by Coulomb's force or Gauss' law is embodied in Weber's force law. Weber devised his force law in order to derive Ampère's force between current elements. But it was basically from this force law of Ampère [1], that Maxwell himself derived for the first time, in 1856, 20 years after Ampère's death, what is known as "Ampère's" circuital law (one of Maxwell's equations), see ref. 2, vol. 1, pp. 242–245. Maxwell, and not Ampère, was the first to derive this law even without the term in the displacement current. Perhaps this is the reason why Maxwell said that Ampère's force between current elements [1], "must always remain the cardinal [most important] formula of electrodynamics" (ref. 17, vol. 2, article 528, p. 175). So Weber's force, by yielding Ampère's law, can be seen to comply with Mawell's equations, as Maxwell showed and emphasized more than once. This is even more evident if we remember that Weber also succeeded in deriving Faraday's law of induction (1831) from his force law. A simple proof of this fact can also be found in Maxwell's *A Treatise on Electricity and Magnetism* where he says:

> After deducing from Ampère's formula for the action [force] between the elements of current, his own formula for the action [force] between moving electric particles, Weber proceeded to apply his formula to the explanation of the production of electric currents by magneto-electric induction. In this he was eminently successful, and we shall indicate the method by which the laws of induced currents may be deduced from Weber's formula." (ref. 17, vol. 2, article 856, p. 486.)

But if Weber's force is so powerful why is it not found nowadays in almost any textbook? There seems to be three main reasons for this. The first one is that Weber derived his force from Ampère's expression [1], assuming Fechner's hypothesis (1845) according to which the usual metallic currents were composed of positive charges moving in one direction relative to the metal or lattice, and of an equal amount of negative charges moving in the opposite direction with the same velocity. At the time this was the simplest hypothesis to be utilized, but since the discovery of electrons at the turn of the century and the realization that only they are responsible for the current, while the positive ions remain fixed in the lattice, the Fechner hypothesis was thought to be a flaw in Weber's theory. But recently we showed that if we assume only Weber's force and the charge neutrality of the current elements, we can still derive Ampère's force [1] for any velocity of the electrons and for any independent velocity of the positive ions [20]. Moreover this will hold true not only when we have stationary ions and mobile electrons (as in metallic currents), but also when the positive ions and the electrons move in opposite directions with different velocities (as in the gaseous plasmas of tokamaks), and even when the positive and or negative charges are accelerated.

The second reason was Helmholtz's critism of Weber's law (17, 21), where he showed that Weber's potential could lead to a "negative mass behaviour" of the charges in some situations such as when they were located in regions of high electrostatic potential. In particular this could make these charges attain velocities greater than that of light, which was never observed. The first to overcome Helmholtz criticism seems to have been Phipps when he proposed that Weber's law is only an approximation valid up to second order in $\dot{r}/c$, inclusively, and suggested an improved form for the generalized potential energy, namely (22, 23):

$$[4] \quad U_p \equiv \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r_{12}}\sqrt{1 - \frac{\dot{r}_{12}^2}{c^2}}$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r_{12}}\left(1 - \frac{\dot{r}_{12}^2}{2c^2} - \frac{\dot{r}_{12}^4}{8c^4} - \frac{\dot{r}_{12}^6}{16c^6} - \dots\right)$$

Phipps' potential reduces to Weber's potential for low velocities. As it is free of the negative mass behaviour it has overcome the limitation pointed out by Helmholtz.

The third and main reason for the neglect of Weber's law in the first half of this century was the success of the electromagnetic theory of light after the experiments of Hertz (1885–1889), which showed the finite velocity of propagation of the electromagnetic effects. Weber's force belongs to the class of action-at-a-distance theories, like Newton's law of gravitation. This means that in these theories if one of the particles changes position slightly the other particle will immediately feel an increase or decrease in the force, no matter how far it is from the first one. The first to overcome this limitation within Weber's theory were Moon and Spencer when they introduced retarded time

($t - r_{12}/c$ instead of t) in Weber's model (24). Recently this approach was further generalized and expanded by Wesley when he showed that Weber's force with time retardation yields the wave equations for the scalar-electric and vector-magnetic potentials (25, 26). This seems to be a promising line of research but it must be emphasized here that this is not the only way of getting finite velocities of propagation for the effects within the model. In the first place it should be remembered that Weber's law by itself already models a delay in the propagation of interactions, as was discussed by Sokol'skii and Sadovnikov when they applied a Weber's force law for gravitation to study the stability of planetary orbits (27). On the other hand it should be stressed that the first to obtain a wave equation for the propagation of an electric perturbation (a pulse of current or voltage, for instance) in a metallic circuit were Kirchhoff and Weber, in 1857, which was therefore previous to Maxwell's equations in their complete form (1860–1864). Both Kirchhoff and Weber worked with Weber's action-at-a-distance theory coupled with the law for the conservation of charges. In particular they showed, working independently of one another that in a wire of negligible resistance the electric disturbance will be propagated along this wire with the velocity $c = (\varepsilon_0\mu_0)^{-1/2}$, (refs. 2, vol. 1, pp. 224–236; 16; 18, vol. 2, pp. 523–535; 28–30). Obviously it is not yet clear how this could work in a vacuum, where there is no material medium (such as the metallic circuit above) to propagate the signal, but their early accomplishment should be kept in mind.

Hertz's experiments have usually been regarded as the definitive confirmation of Maxwell's theory. Can they be explained with Weber's electrodynamics? To our knowledge these experiments were never analysed from a point of view based on Weber's law. So our answer to this question is that we do not know. On the other hand it is known that Maxwell's theory is not the only one compatible with Hertz's experiments. For instance, Ritz's ballistic theory has been proved to be equally consistent with them (ref. 18, vol. 1, pp. 230–233 and vol. 2, pp. 499–512). But even if Weber's electrodynamics also proves to be compatible with them, it will need to face other challenges. Since Hertz's experiments, there have been an overwhelming number of experiments confirming Maxwell's theory of electromagnetism in many different aspects. Can Weber's theory or a suitable modification of it stand up to the same level of experimental scrutiny? Once more, we do not know yet. After a certain neglect in the first half of this century, Weber's electrodynamics and extensions of it have been researched extensively in recent years. We need to wait some time before these modern developments are analysed from different perspectives. But any theory of Weber's type will need to face the challenge of being compatible with a large number of experimental results to be of real scientific value, and not only of historical interest. These are the three main reasons for the neglect to Weber's theory, and how they have been overcome in recent times.

Now we would like to discuss briefly some recent experiments and ideas that have once more brought Weber's law to the forefront of modern sicence. First of all a great analogy has been noted between the structure of Weber's force and those describing nuclear interactions (31). Although it would be quite interesting to discuss this subject here, as in this paper we are dealing with the unification of the forces of nature, this would be beyond the scope of this work as our analysis here is restricted to electromagnetic and gravitational theories.

If Weber's force describes correctly the interaction between electric charges at least for low velocities, then a stationary and neutral electric circuit carrying a constant current should exert a force on a stationary charge brought nearby. In 1976 Edwards *et al.* found such a force, and the direction and order of magnitude that they detected was in agreement with what could be expected according to Weber's model (10). However the following year Bartlett and Ward did not find such a force (32), and in 1985 Sansbury (33) also obtained a force on a stationary charge due to a steady and neutral current, but in the opposite direction than that obtained by Edwards *et al.* It should be noted in these works they did not repeat the experiments of each another, but all of them were trying to test the same general idea. As we have already analysed these completely opposite findings and their relation to Weber's electrodynamics elsewhere (34), we will not discuss this subject here again. In our previous paper (34), we also discussed the relevance of this problem to plasma physics. Clearly more experimental results are needed along these lines, some of which (35–38) are yet to be analysed from Weber's point of view.

Another kind of experiment is related to Ampère's force [1]. As happened with Weber's force, it is difficult to find [1] in any textbook. Instead we find only Grassmann's force (1845), according to which the force of $I_2\,dl_2$ on $I_1\,dl_1$ is given by

$$[5] \quad d^2\boldsymbol{F}_{21} \equiv I_1\,d\boldsymbol{l}_1 \times d\boldsymbol{B}_2$$

$$= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\left[(d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2)\hat{\boldsymbol{r}}_{12} - (d\boldsymbol{l}_1\cdot\hat{\boldsymbol{r}}_{12})\,d\boldsymbol{l}_2\right]$$

In this expression $d\boldsymbol{B}_2$ is the magnetic field as given by Biot–Savart's law of 1820, namely,

$$d\boldsymbol{B}_2 \equiv \frac{\mu_0(I_2\,d\boldsymbol{l}_2 \times \hat{\boldsymbol{r}}_{12})}{4\pi r_{12}^2}$$

While Ampère's force [1] always follows Newton's third law, it is only valid for Grassmann's force [5] in some particular situations. However when we have two or more closed circuits it is a known fact that Grassmann's force law will not only comply with Newton's action and reaction law but will also predict the same forces between the circuits as Ampère's force law [1] (39). On the other hand if we have a single closed circuit and calculate the force between part of this circuit and the remainder of the same circuit the two force laws do not seem to agree with one another. Experiments performed in the last 10 years with a single closed circuit (40–46) are in complete quantitative agreement with Ampère's force law [1]. Up to now it is not yet completely clear if they can be equally well explained by Grassmann's force law, and there is a lively discussion in the literature (25, 26, 47–51) where two points of view have been put forward:

(*i*) Ampère's force is the only one in full agreement with the experimental findings, and

(*ii*) even for a single closed circuit, Grassmann's force will predict exactly the same results as Ampère's force so that both of them would be compatible with the facts.

The relevance of this discussion to Weber's electrodynamics is that with Weber's force we derive only Ampère's force between current elements, but not Grassmann's force. So it would be extremely important to settle this problem experimentally in order to decide the question. It is worth while remembering that Maxwell knew Grassmann's force (ref. 17, vol. 2, article 526, p. 174) and made the following comparison between Ampère's force [1], Grassmann's force [5], and two other force laws of his own, between current elements:

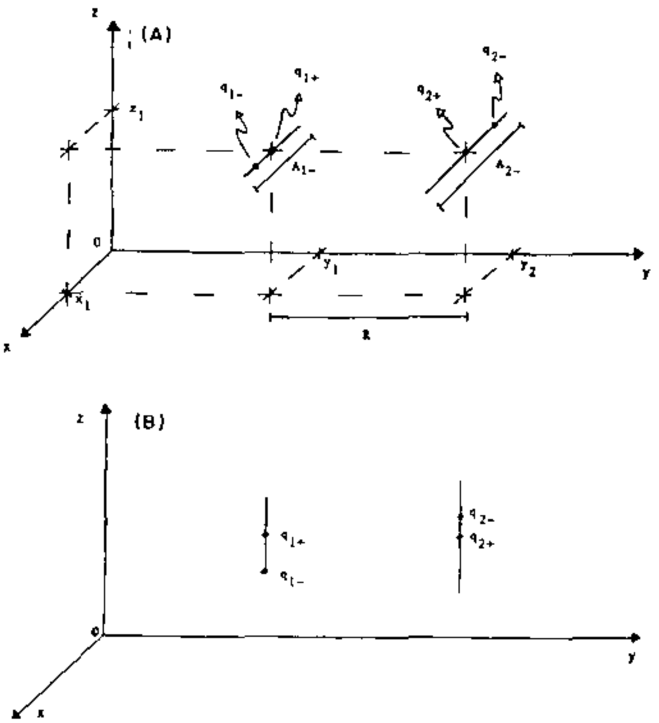

FIG. 1. Two neutral dipoles separated along the y axis. The negative charge of dipole 1, q_{1-}, oscillates around the positive one, q_{1+}, with frequency ω_1, and amplitude of oscillation A_{1-}. The negative charge of dipole 2, q_{2-}, oscillates around the positive one, q_{2+}, with frequency ω_2, and amplitude of oscillation A_{2-}. (A) q_{1-} and q_{2-} oscillating along the x axis. (B) q_{1-} and q_{2-} oscillating along the z axis.

> Of these four different assumptions that of Ampère is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them. (Ref. 17, vol. 2 article 527, p. 174)

But even if it is found that Ampère's force is the correct one this does not mean that Weber's force is exact. As we have shown, it cannot be applied to charges moving near the velocity of light as it leads to results not borne out by experiment (52, 53). Moreover, as we have seen, to overcome Helmholtz criticism of Weber's law Phipps had to modify Weber's potential for high velocities. Wesley's idea of introducing time retardation in Weber's law will essentially reduce in higher order corrections to Weber's theory by introducing terms of the order $\dot{r}_{12}^4/c^4$ and higher. With all of this we can only conclude that Weber's electrodynamics is only an approximation valid up to second order in $\dot{r}_{12}/c$, inclusive. For charges moving at high velocities the model needs to be modified to correctly describe their behaviour. And this brings us back to the main subject of this paper, as the model, which we utilize here to derive gravitation from electromagnetism, is a generalization of Weber's law including terms of fourth and higher orders in $\dot{r}_{12}/c$. The model is described in the next section.

3. The model

As we want to derive gravitation from electromagnetism we should begin with only electromagnetic forces. Our main assumption, following the discussion of the previous section, is that the generalized potential energy between charges q_1 and q_2 is given by

$$[6]\quad U = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{1}{r_{12}}\left[1 - \alpha\left(\frac{\dot{r}_{12}}{c}\right)^2 - \beta\left(\frac{\dot{r}_{12}}{c}\right)^4 - \gamma\left(\frac{\dot{r}_{12}}{c}\right)^6 - \dots\right]$$

With the exception of the numerical constants α, β, γ, ..., all the other quantities in this expression have already been defined. Weber's energy [3] is a special case of this expression when $\alpha = 1/2$ and $\beta = \gamma = \dots = 0$. To maintain his results we keep this value of α but suppose $\beta \neq 0$, $\gamma \neq 0$, etc. Essentially, we are assuming here that Weber's electrodynamics is a good approximation for low velocities, which must be modified to correctly describe the behaviour of charges at high velocities, radiation phenomena, etc. One of our goals here is also to begin the determination of the values of β, γ, etc.

Assuming as usual that the force exerted by q_2 on q_1 can be given by the expression

$$F_{21} = -\hat{r}_{12}\frac{dU}{dr_{12}}$$

or that

$$v_{12}\cdot F_{21} = \frac{-dU}{dt}$$

yields,

$$[7]\quad F_{21} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\hat{r}_{12}}{r_{12}^2}\left(1 - \alpha\frac{\dot{r}_{12}^2 - 2r_{12}\ddot{r}_{12}}{c^2} - \beta\frac{\dot{r}_{12}^4 - 4\dot{r}_{12}^2r_{12}\ddot{r}_{12}}{c^4} - \gamma\frac{\dot{r}_{12}^6 - 6\dot{r}_{12}^4r_{12}\ddot{r}_{12}}{c^6} - \dots\right)$$

Essentially [6] and [7] are our main assumptions. Phipps' potential [4], for instance, is also a special case of [6] with $\alpha = 1/2$, $\beta = 1/8$, $\gamma = 1/16$, ... Although Phipps succeeded in overcoming Helmholtz criticism of Weber's law, his potential is obviously not the only one that can do that. This is the reason why we want to work with a more general expression for the potential, instead of dealing with only a very specific one. From our previous discussion it is clear that [6] and [7] are also compatible with all the conservation laws of classical physics, so that our starting point is a reasonable one.

4. Gravitation as a fourth-order electromagnetic effect

The general idea is to calculate, using [7], the force between two neutral dipoles. Each dipole is supposed to consist of a positive charge at the center and a negative charge oscillating harmonically around the positive charge, as usual. Each dipole is allowed to move as a whole and we calculate the force between the dipoles in relative motion. We represent the positive and negative charge of dipole 1 by q_{1+} and q_{1-}, respectively, and those of dipole 2 by q_{2+} and q_{2-}.

In our first situation (Fig. 1A) we have

$$[8]\quad r_1 = [x_1(t) + A_1\sin(\omega_1t + \theta_1)]\,\hat{x} + y_1(t)\hat{y} + z_1(t)\hat{z}$$

$$[9]\quad r_2 = [x_1(t) + A_2\sin(\omega_2t + \theta_2)]\,\hat{x} + y_2(t)\hat{y} + z_1(t)\hat{z}$$

In these expressions $r_1(t)$ and $r_2(t)$ are the positions of a charge in dipoles 1 and 2, respectively, A_1 and A_2 are the amplitudes of oscillation of the negative charges around the equilibrium positions ($A_{1+} = A_{2+} = 0$, $A_{1-} \neq 0$, and $A_{2-} \neq 0$), ω_1 and ω_2 are the frequencies of oscillation of the negative charges, and θ_1 and θ_2 their phases of oscillation. This means that in the first situation shown Fig. 1A we choose the axes of the coordinates so that the equilibrium position along the x axis of the two dipoles is equal, the same being valid for the z axis (later on we will generalize this condition). In this first situation the two negative charges are oscillating along the x axis, but obviously later on we will perform an average including all directions of oscillation. We allow different frequencies of vibration and also different phases in each dipole, which are the most natural hypotheses to be utilized.

Now, we define the relative position, velocity, and acceleration between the centers of the dipoles by

$$[10] \quad \boldsymbol{R} \equiv (x_1 - x_2)\,\hat{x} + (y_1 - y_2)\,\hat{y} + (z_1 - z_2)\,\hat{z} \equiv R_x\hat{x} + R_y\hat{y} + R_z\hat{z}$$

$$[11] \quad \boldsymbol{V} \equiv \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} \equiv V_x\,\hat{x} + V_y\,\hat{y} + V_z\,\hat{z}$$

$$[12] \quad \boldsymbol{A} \equiv \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{R}}{\mathrm{d}t^2} \equiv A_x\,\hat{x} + A_y\,\hat{y} + A_z\,\hat{z}$$

So that from [8] and [9] we have

$$[13] \quad \boldsymbol{r}_{12} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2 = B_0\,\hat{x} + R_y\,\hat{y}$$

$$[14] \quad \boldsymbol{v}_{12} \equiv \frac{\mathrm{d}\boldsymbol{r}_{12}}{\mathrm{d}t} \equiv \boldsymbol{v}_1 - \boldsymbol{v}_2 = B_1\hat{x} + V_y\,\hat{y}$$

$$[15] \quad \boldsymbol{a}_{12} \equiv \frac{\mathrm{d}^2\boldsymbol{r}_{12}}{\mathrm{d}t^2} \equiv \boldsymbol{a}_1 - \boldsymbol{a}_2 = B_2\,\hat{x} + A_y\,\hat{y}$$

where

$$[16] \quad B_0 \equiv A_1\sin(\omega_1 t + \theta_1) - A_2\sin(\omega_2 t + \theta_2)$$

$$[17] \quad B_1 \equiv A_1\omega_1\cos(\omega_1 t + \theta_1) - A_2\omega_2\cos(\omega_2 t + \theta_2)$$

$$[18] \quad B_2 \equiv -A_1\omega_1^2\sin(\omega_1 t + \theta_1) + A_2\omega_2^2\sin(\omega_2 t + \theta_2)$$

To calculate the force exerted by q_2 on q_1 in this situation and in the following ones, we assume that (defining the distance between the dipoles by $R \equiv |\boldsymbol{R}| = (\boldsymbol{R}\cdot\boldsymbol{R})^{1/2}$):

$$[19] \quad \frac{A_1^2}{R^2} \ll 1, \qquad \frac{A_2^2}{R^2} \ll 1$$

$$[20] \quad R^2 \gg \frac{c^2}{\omega_1^2}, \qquad R^2 \gg \frac{c^2}{\omega_2^2}$$

Later on we will justify these approximations numerically, but we can say now that they are quite reasonable indicating that the amplitudes of oscillation of the microscopic dipoles are much smaller than the distance between them and that their distance is also much greater than c/ω (we are considering effects only in the far zone). If there is a relative velocity and acceleration between the dipoles we also assume that (defining $V \equiv |\boldsymbol{V}| = (\boldsymbol{V}\cdot\boldsymbol{V})^{1/2}$, and $A \equiv |\boldsymbol{A}| = (\boldsymbol{A}\cdot\boldsymbol{A})^{1/2}$):

$$[21] \quad R^2\omega_1^2 \gg V^2 \gg A_1^2\omega_1^2$$

$$[22] \quad R^2\omega_2^2 \gg V^2 \gg A_2^2\omega_2^2$$

$$[23] \quad R^2\omega_1^2 \gg RA \gg A_1^2\omega_1^2$$

$$[24] \quad R^2\omega_2^2 \gg RA \gg A_2^2\omega_2^2$$

These approximations will also be justified numerically in the next section.

Applying [10]–[24] in [7] yields, up to the sixth order in $1/c$,

$$\begin{aligned} \boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\boldsymbol{R}}{R^3}\Bigg\{1 &- \frac{\alpha}{c^2}\left[\frac{(\boldsymbol{R}\cdot\boldsymbol{V})^2}{R^2} - 2R\ddot{R} - 4V_xB_1 - 2A_xB_0 - 2B_1^2 - 2B_0B_2\right] \\ &- \frac{\beta}{c^4}\left[\frac{(\boldsymbol{R}\cdot\boldsymbol{V})^4}{R^4} - \frac{4(\boldsymbol{R}\cdot\boldsymbol{V})^2}{R^2}(R\ddot{R} + 2V_xB_1 + A_xB_0 + B_1^2 + B_0B_2) - \frac{8(\boldsymbol{R}\cdot\boldsymbol{V})V_xB_0R\ddot{R}}{R^2}\right] \\ &- \frac{\gamma}{c^6}\left[\frac{(\boldsymbol{R}\cdot\boldsymbol{V})^6}{R^6} - \frac{6(\boldsymbol{R}\cdot\boldsymbol{V})^4}{R^4}(R\ddot{R} + 2V_xB_1 + A_xB_0 + B_1^2 + B_0B_2) - \frac{24(\boldsymbol{R}\cdot\boldsymbol{V})^3V_xB_0R\ddot{R}}{R^4}\right]\Bigg\} \\ &+ \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\hat{x}}{R^2}\left[\frac{2\alpha\ddot{R}B_0}{c^2} + \frac{4\beta(\boldsymbol{R}\cdot\boldsymbol{V})^2\ddot{R}B_0}{c^4R^2} + \frac{6\gamma(\boldsymbol{R}\cdot\boldsymbol{V})^4\ddot{R}B_0}{c^6R^4}\right] \end{aligned}$$

where

$$\dot{R} = \frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\boldsymbol{R}\cdot\boldsymbol{V}}{R} \qquad \text{and} \qquad \ddot{R} \equiv \frac{\mathrm{d}^2R}{\mathrm{d}t^2} = \frac{\mathrm{d}\dot{R}}{\mathrm{d}t} = \frac{\{\boldsymbol{V}\cdot\boldsymbol{V} - [(\boldsymbol{R}\cdot\boldsymbol{V})^2/R^2] + \boldsymbol{R}\cdot\boldsymbol{A}\}}{R}$$

To calculate the force between the dipoles in this situation we utilize the fact that

$$A_{1+} = A_{2+} = 0, \qquad A_{1-} \neq 0, \qquad A_{2-} \neq 0$$

and that their charge neutrality is $q_{1-} = -q_{1+}$ and $q_{2-} = -q_{2+}$. The force between the dipoles is the sum of four terms, namely, the force of q_{2+} on q_{1+} and q_{1-}, and the force of q_{2-} on q_{1+} and q_{1-}. It can be expressed as

$$\boldsymbol{F} = \boldsymbol{F}_{2+,1+} + \boldsymbol{F}_{2+,1-} + \boldsymbol{F}_{2-,1+} + \boldsymbol{F}_{2-,1-}$$

We are interested only in an average effect. We perform three averages, two of which are in phases θ_1 and θ_2, allowing any value between zero and 2π. This is equivalent to a realistic situation in which we have many dipoles each of which has a different phase at the same time t. Then we perform an average in time. To do this we suppose that $\omega_1 = n\omega_2$, where n is a positive integer, and integrate the force from $t = 0$ to the highest period $t = T_2 = 2\pi/\omega_2$, dividing the result by the period T_2 to get an average value. This hypothesis of multiple frequencies is utilized to facilitate the calculations, but it is not essential for the results. For instance, if the frequencies are not multiple to one another we can integrate from $t = 0$ to $t = T$, divide the result by T, and then study the limit when $T \to \infty$. Moreover we suppose that the relative motion between the dipoles, if any, is such that $\boldsymbol{R}$, $\boldsymbol{V}$, and $\boldsymbol{A}$ can be considered as constants between $t = 0$ and $t = T_2$. This is an usual situation, as we will see in the following.

Calculating the sum of the four terms ($\boldsymbol{F}_{2+,1+}$, etc.) with the previous conditions to get the force between two neutral dipoles and then performing the three averages as indicated above, we find from [25] that the resultant average force exerted by dipole 2 on dipole 1 in this first situation ([8] and [9], or Fig. 1A) is zero (at least up to the sixth order in $1/c$, inclusively). By symmetry the same will happen when the negative charges of both dipoles oscillate along the z axis, and the dipoles are separated only along the y axis, as indicated in Figure 1B.

We now consider another situation (Fig. 2):

$$[26] \quad \boldsymbol{r}_1 = x_1(t)\hat{x} + [y_1(t) + A_1 \sin(\omega_1 t + \theta_1)]\hat{y} + z_1(t)\hat{z}$$

$$[27] \quad \boldsymbol{r}_2 = x_1(t)\hat{x} + [y_2(t) + A_2 \sin(\omega_2 t + \theta_2)]\hat{y} + z_1(t)\hat{z}$$

Again we choose the coordinate system so that the dipoles are separated only along the y axis, but now the negative charges in both of them are allowed to oscillate only along this direction. Calculating [7] up to the sixth order in this situation yields

$$[28] \quad \boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{R_y\hat{y}}{|R_y|^3}\Bigg\{1 - \frac{\alpha}{c^2}\Big[3V_y^2 - 2\boldsymbol{V}\cdot\boldsymbol{V} - 2R_yA_y + 2V_yB_1 - 2A_yB_0 - 2R_yB_2 + B_1^2 - 2B_0B_2 + 4A_yB_0 + 4B_0B_2\Big]$$

$$- \frac{\beta}{c^4}\Big[V_y^4 - 4V_y^2R\ddot{R} + 4V_y^3B_1 - 4V_y^2A_yB_0 - 4R_yV_y^2B_2 - 8V_yR\ddot{R}B_1 + 6V_y^2B_1^2 - 4V_y^2B_0B_2 - 8V_yA_yB_0B_1$$

$$- 8R_yV_yB_1B_2 - 4R\ddot{R}B_1^2 + 4V_yB_1^3 - 8V_yB_0B_1B_2 - 4A_yB_0B_1^2 - 4R_yB_1^2B_2 + B_1^4 - 4B_0B_1^2B_2 + 8V_y^2A_yB_0 + 8V_y^2B_0B_2$$

$$+ 16V_yA_yB_0B_1 + 16V_yB_0B_1B_2 + 8A_yB_0B_1^2 + 8B_0B_1^2B_2\Big] - \frac{\gamma}{c^6}\Big[V_y^6 - 6V_y^4R\ddot{R} + 6V_y^5B_1 - 6V_y^4A_yB_0 - 6R_yV_y^4B_2$$

$$- 24V_y^3R\ddot{R}B_1 + 15V_y^4B_1^2 - 6V_y^4B_0B_2 - 24V_y^3A_yB_0B_1 - 24R_yV_y^3B_1B_2 - 36V_y^2R\ddot{R}B_1^2 + 20V_y^3B_1^3 - 24V_y^3B_0B_1B_2$$

$$- 36V_y^2A_yB_0B_1^2 - 36R_yV_y^2B_1^2B_2 - 24V_yR\ddot{R}B_1^3 + 15V_y^2B_1^4 - 36V_y^2B_0B_1^2B_2 - 24V_yA_yB_0B_1^3 - 24R_yV_yB_1^3B_2 - 6R\ddot{R}B_1^4$$

$$+ 6V_yB_1^5 - 24V_yB_0B_1^3B_2 - 6A_yB_0B_1^4 - 6R_yB_1^4B_2 + B_1^6 - 6B_0B_1^4B_2 + 12V_y^4A_yB_0 + 12V_y^4B_0B_2 + 48V_y^3A_yB_0B_1$$

$$+ 48V_y^3B_0B_1B_2 + 72V_y^2A_yB_0B_1^2 + 72V_y^2B_0B_1^2B_2 + 48V_yA_yB_0B_1^3 + 48V_yB_0B_1^3B_2 + 12A_yB_0B_1^4 + 12B_0B_1^4B_2\Big]\Bigg\}$$

Here $R\ddot{R} = \boldsymbol{V}\cdot\boldsymbol{V} - V_y^2 + R_yA_y$.

The sum of the four terms and the triple average as given above yields for the force of dipole 2 on dipole 1 the result

$$[29] \quad \boldsymbol{F}_{21} = \frac{q_{1+}q_{2+}}{4\pi\varepsilon_0}\frac{R_y\hat{y}}{|R_y|^3}\frac{A_{1-}^2\omega_1^2A_{2-}^2\omega_2^2}{c^4}\left[\frac{\beta}{2} - \frac{\gamma}{2}\frac{27V_y^2 - 18(\boldsymbol{V}\cdot\boldsymbol{V} + R_yA_y)}{c^2}\right]$$

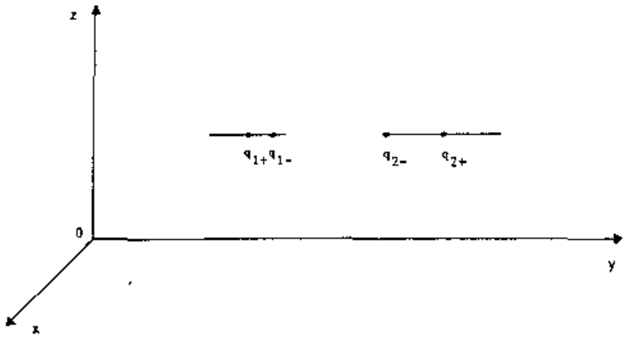

FIG. 2. The same as Fig. 1 but now q_{1-} and q_{2-} oscillating along the y axis.

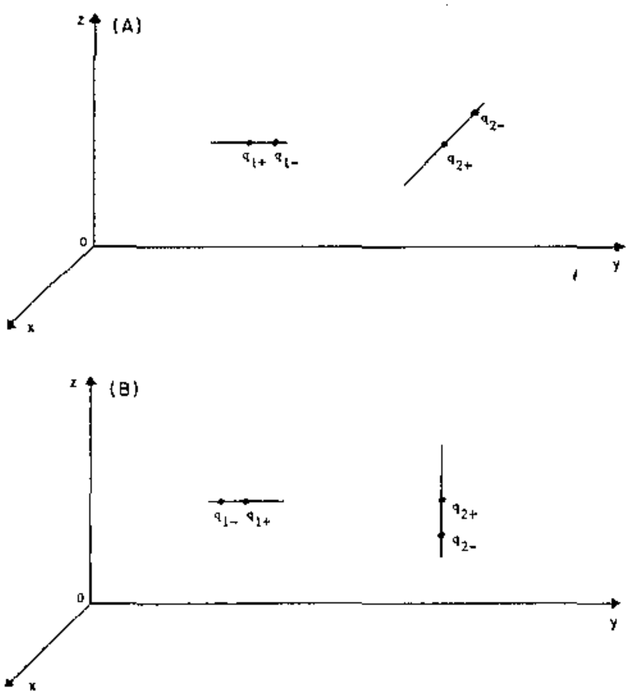

FIG. 3. The same as Fig. 1 but now: (A) q_{1-} oscillating along the y axis and q_{2-} along the x axis. (B) q_{1-} oscillating along the y axis and q_{2-} along the z axis.

Now we do not have a null result any more, instead, we find a resultant average force that appears only on the fourth- and sixth-order terms (probably in higher order terms as well, but here we restrict ourselves to the sixth order).

Another situation occurs when the negative charge of dipole 1 oscillates along the y axis, while the negative charge of dipole 2 oscillates along the x axis, Fig. 3A, so that

$$[30] \quad \boldsymbol{r}_1 = x_1(t)\hat{x} + [y_1(t) + A_1 \sin(\omega_1 t + \theta_1)]\hat{y} + z_1(t)\hat{z}$$

$$[31] \quad \boldsymbol{r}_2 = [x_1(t) + A_2 \sin(\omega_2 t + \theta_2)]\hat{x} + y_2(t)\hat{y} + z_1(t)\hat{z}$$

Following the same procedure as above, we find the result for the average value of the force exerted by dipole 2 on dipole 1 to be

$$[32] \quad \boldsymbol{F}_{21} = -\frac{q_{1+}q_{2+}}{4\pi\varepsilon_0}\frac{R_y\hat{y}}{|R_y|^3}\frac{A^2_{1-}\,\omega_1^2 A^2_{2-}\,\omega_2^2}{c^4}\left(\beta + \frac{9\gamma A^2_{1-}\,\omega_1^2}{8c^2}\right)$$

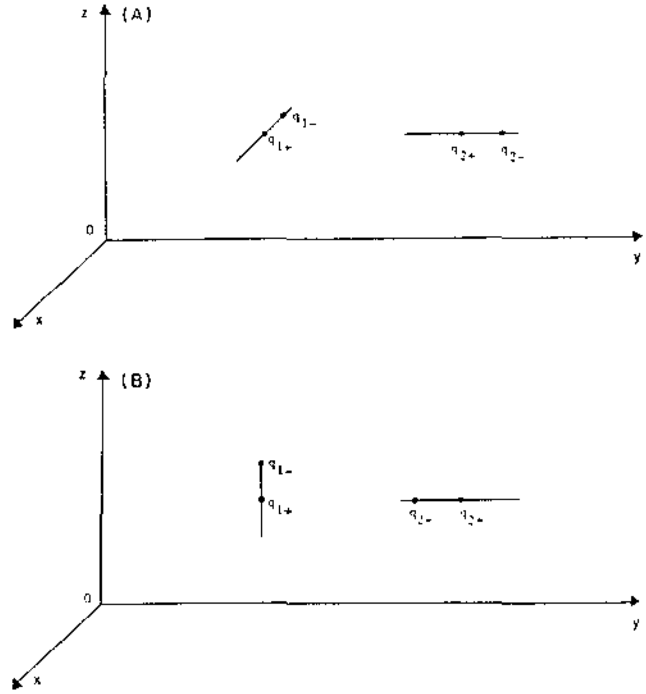

FIG. 4. The same as Fig. 1 but now: (A) q_{1-} oscillating along the x axis and q_{2-} along the y axis. (B) q_{1-} oscillating along the z axis and q_{2-} along the y axis.

By symmetry the same result is found when the negative charge of dipole 1 oscillates along the y axis while the negative charge of dipole 2 oscillates along the z axis, Fig. 3B.

On the other hand, when the negative charge of dipole 2 oscillates along the y axes while the negative charge of dipole 1 oscillates along the x or z axis, Figs. 4A and 4B, we get the same result as in [32], but with $A^2_{1-}\,\omega_1^2$ instead of $A^2_{2-}\,\omega_2^2$, and $A^2_{2-}\,\omega_2^2$ instead of $A^2_{1-}\,\omega_1^2$.

Another situation that is missing occurs when the negative charge of dipole 1 oscillates along the z axis, while the negative charge of dipole 2 oscillates along the x axis, Figs. 5A, namely:

$$[33] \quad \boldsymbol{r}_1 = x_1(t)\hat{x} + y_1(t)\hat{y} + [z_1(t) + A_1 \sin(\omega_1 t + \theta_1)]\hat{z}$$

$$[34] \quad \boldsymbol{r}_2 = [x_1(t) + A_2 \sin(\omega_2 t + \theta_2)]\hat{x} + y_2(t)\hat{y} + z_1(t)\hat{z}$$

Performing the same procedure as above yields for the average force of dipole 2 on dipole 1 a zero value. The same happens when the negative charge of dipole 1 oscillates in the x direction while the negative charge of dipole 2 oscillates in the z direction, Fig. 5B.

The reason why we obtained a zero value here while in the situations shown in Figs. 3 and 4 we obtained a value other than zero is that here the negative charges of both dipoles oscillate in directions orthogonal to one another and also orthogonal to the line joining the dipoles. On the other hand, in the situations shown in Figs. 3 and 4 although the negative charges of both dipoles oscillate in directions orthogonal to one another, one of them oscillates along the line joining the dipoles. This is the reason for the different results between the situations shown in Figs. 3 and 4, and that shown in Fig. 5.

To find the average value for the force exerted by dipole 2 on dipole 1 we need to perform a fourth average. That is, we need to add the results of these nine cardinal situations (the

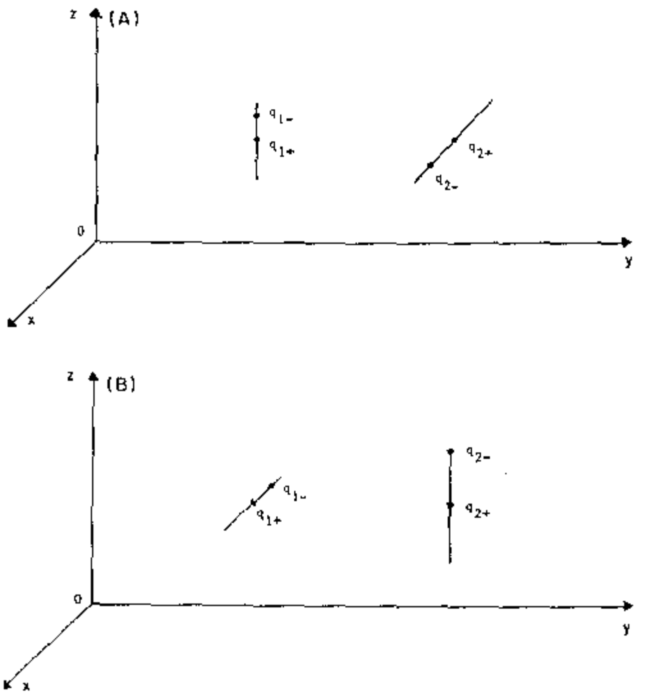

FIG. 5. The same as Fig. 1 but now: (A) q_{1-} oscillating along the z axis and q_{2-} along the x axis. (B) q_{1-} oscillating along the x axis and q_{2-} along the z axis.

negative charge of dipole 1 oscillating along x, y, and z combined with the negative charge of dipole 2 oscillating along x, y, and z, see Figs. 1–5) and then divide the result by nine. Utilizing [21]–[24] once more and generalizing the result for dipoles 1 and 2 located anywhere in space (and not only separated along the y axis) yields the final averaged result

$$[35] \quad \boldsymbol{F}_{12} = -\frac{7\beta}{18}\frac{q_{1+}q_{2+}}{4\pi\varepsilon_0}\frac{\boldsymbol{R}}{R^3}\frac{A_{1-}^2\omega_1^2A_{2-}^2\omega_2^2}{c^4} \times \left(1 + \frac{\gamma}{\beta}\frac{45\dot{R}^2 - 18R\ddot{R}}{7c^2}\right)$$

where $\dot{R} \equiv \mathrm{d}R/\mathrm{d}t$ and $\ddot{R} \equiv \mathrm{d}^2R/\mathrm{d}t^2$.

5. General discussion and conclusions

We consider first the term that falls as c^{-4} in [35]. If β is a positive number, this term represents an attractive force exactly like that of gravitation, which falls as $1/R^2$; is along the line joining the particles (in this case these "particles" are two small and neutral dipoles in which the negative charges oscillate around the equilibrium positions); and follows Newton's action and reaction law. As an example of a potential that could give rise to such a force we have that of Phipps given by [4], where $\beta = 1/8$.

To have an idea of the order of magnitude of these terms we suppose $q_{1+} = q_{2+} = e$, where $-e$ is the electron's charge, and $A_{1-} \simeq A_{2-} \simeq 10^{-10}$ m (typical size of an atom or molecule where the electrons are vibrating around a positive nucleus). To simulate Newton's gravitational force we must have (with $\omega_1 = \omega_2 \equiv \omega$ and $\beta = 1/8$):

$$[36] \quad \frac{7}{18}\frac{1}{8}\frac{e^2}{4\pi\varepsilon_0}\frac{A_{1-}^4\omega^4}{c^4} = GM^2$$

where M is what we call the "mass" of the neutron or of the hydrogen atom. This implies that $\omega \simeq 6 \times 10^9\ \mathrm{s}^{-1}$. This microwave frequency is exactly the kind of frequency we have on the atomic scale. So, with amplitudes and oscillating frequencies like this, we can reproduce Newton's law of gravitation as a fourth-order electromagnetic effect. Moreover, with this range of frequencies we usually satisfy [19] and [20] because $A_1^2 \simeq A_2^2 \simeq 10^{-20}\ \mathrm{m}^2$ and $c^2/\omega^2 \simeq 2.5 \times 10^{-3}\ \mathrm{m}^2$. Usually we deal in gravitation with distances R spanning from 1–10^{20} m (typical size of galaxy); [19] and [20] are automatically satisfied. The period of oscillation is typically of the order $T = 2\pi/\omega \simeq 10^{-9}$ s. This shows that our approximation of considering $\boldsymbol{R}$, $\boldsymbol{V}$, and $\boldsymbol{A}$ as essentially constant during the time average from $t = 0$ to $t = T$ is also easily justifiable owing to the extremely short value of T. Y so

We now analyse the sixth-order term in [35]. This is another term that follows Newton's action and reaction law and is along the line joining the two particles (this will happen in all orders as we began with a generalization of Weber's force law, which complies with Newton's third law in the strong form). From our previous analysis and considering that we have N_1 dipoles around the position

$$\boldsymbol{R}_1 \equiv x_1\hat{x} + y_1\hat{y} + z_1\hat{z}$$

and N_2 dipoles around

$$\boldsymbol{R}_2 \equiv x_2\hat{x} + y_2\hat{y} + z_1\hat{z}$$

we can write [35] as

$$[37] \quad \boldsymbol{F}_{21} = -GM_1M_2\frac{\boldsymbol{R}}{R^3}\left(1 + \frac{18\gamma}{\beta}\frac{2.5\dot{R}^2 - R\ddot{R}}{7c^2}\right)$$

when M_1 and M_2 are the "masses" of the group of dipoles 1 and 2, respectively, so that

$$[38] \quad GM_1M_2 = \frac{7\beta N_1q_{1+}A_{1-}^2\omega_1^2N_2q_{2+}A_{2-}^2\omega_2^2}{72\pi\varepsilon_0c^4}$$

Recently we utilized an expression similar to [37] as a model for the gravitational interaction between material particles [54]. In this previous model, it was possible to implement Mach's principle quantitatively and we could also derive the proportionality between inertial and gravitational masses. It is a known fact that in a model like this it is possible to derive the precession of the perihelion of the planets through an orbit equation different from that of general relativity, but yielding the same algebraic result for the precession, and agreeing with the observed values (54–56). Equation [37] of this work is slightly different from eq. [1] of ref. 54, but both force laws will essentially give the same results provided $\gamma/\beta < 0$ (this yields an equivalent to Newton's second law of motion, and then the proportionality of inertia and weight) and $-18\gamma/7\beta = 6$ (this yields the correct value of the precession of the perihelion of the planets, and to show this, it is only necessary to follow a standard procedure like that presented in ref. 54). The main conditions required to obtain Newton's law of universal gravitation and to keep the results of ref. 54 are then

$$[39] \quad \beta > 0 \quad \text{and} \quad \frac{\gamma}{\beta} = -\frac{7}{3}$$

It can easily be seen that we cannot have a generalized potential like the one expressed by $(q_1q_2/(4\pi\varepsilon_0\ r_{12}))\ (1 + a\dot{r}^2/c^2)^b$, which satisfies [39] and [6], if we, moreover, impose that $ab = -1/2$ in order to generate Weber's potential in second order. On the other hand, there are many functions of other forms that can be expanded in the same way as [6] and which satisfy [39] and $\alpha = 1/2$. Obviously we cannot fix the form of the function giving only some properties of its first three coefficients. For this reason we restrict ourselves to $\alpha = 1/2$, [39] and to general expressions such as [6] and [7].

We only applied the sixth-order terms in [35] to calculate the precession of the perihelion (54). We now justify approximations [21]–[24] for this case, using the planet Mercury for the calculations in its interaction with the Sun, as it has the greatest value for precession in the Solar System: $R \simeq 6 \times 10^{10}$ m, $V \simeq 5 \times 10^4$ m s^{-1}, and $A \simeq V^2/R \simeq 4 \times 10^{-2}$ m s^{-2}. With the previous values of $A_{1-} \simeq A_{2-} \simeq 10^{-10}$ m and $\omega_1 \simeq \omega_2 \simeq 6 \times 10^9$ s^{-1} we get $R^2\omega_1^2 \simeq 10^{41}$ m^2 s^{-2}, $RA \simeq V^2 \simeq 10^9$ m^2 s^{-2}, $A_{1-}^2\omega_1^2 \simeq 10^{-1}$ m^2 s^{-2}. These values are in complete agreement with [21]–[24], justifying our whole procedure.

In conclusion we may say that in this model of generalized Weber electrodynamics we obtain: electrostatics as a zeroth-order effect, magnetism and Faraday's induction as a second-order effect, gravitation as a fourth-order effect, and inertia and precession of the perihelion as a sixth-order electromagnetic effect. Of course if we go to the eighth and higher orders we can obtain other relevant effects but in this paper we restrict ourselves to the sixth order.

Recently Dragone tried to derive gravitation from electromagnetism following the general idea of considering a force between neutral dipoles (57). The differences with our work can be presented here:

(*i*) He utilized a different force law between charges q_1 and q_2.

(*ii*) He went up to only the second order in v/c, while we went to the sixth order and we obtained a force law similar to gravitation at the fourth order. The reason why he could obtain a force law similar to gravitation at the second order in v/c was that he fixed the phases of the two dipoles so as to agree with one another (coherent oscillation). If he had performed an average in θ_1 and (or) in θ_2 as we did here, his force law would go to zero after averaging, as happened with ours at the second order in v/c.

(*iii*) He fixed the frequencies of the two dipoles to be equal to one another, while in our model the frequencies could be different from one another.

(*iv*) We also allowed a relative motion between the dipoles, while he fixed them in the laboratory, $V = 0$ and $A = 0$.

(*v*) Another limitation of his model is that he considered the oscillations of the negative charges only in the situation analogous to our Fig. 1B, while we performed an average over the nine cardinal situations of Figs. 1–5.

To arrive at a coupling between electromagnetic and gravitational interactions a completely different approach has recently been given by Jaakkola (58). In his work he connects in a single theory the cosmological redshift, the screening of the gravitational attraction, the flat rotation curves of galaxies, etc. As the scope of this important work by Jaakkola on the electrogravitational coupling is very different from the one that we are developing here, we will not discuss it further. A detailed comparison between these two approaches would go far beyond the scope of this paper.

Before discussing some possible specific limitations of the model presented here, we would like to mention a general aspect of Weber's electrodynamics: In its present form Weber's theory is nonrelativistic (here we are utilizing the name "relativistic" for any model compatible with the special theory of relativity). On the other hand the present day theories of electromagnetism and gravity are known to be relativistic. Despite the nonrelativistic aspect of this approach, the Weber-type model of this paper led us to gravity like effects. This characteristic of Weber's theory seems to be worthy of consideration.

Now we would like to stress some possible limitations of the model presented in this paper. The first one is that we consider [6] as our starting point, and this potential is an even function of $\dot{r}_{12}/c$. A more general function would also include odd powers of $\dot{r}_{12}/c$, but here we restrict ourselves to [6] not only because Phipps' potential is of this kind but also owing to the fact that in our previous model, which leads to an implementation of Mach's principle, this was the kind of function utilized (54). Another limitation of this model is that we consider the negative charge of dipole 1 oscillating at a frequency ω_1 and the negative charge of dipole 2 oscillating at a frequency ω_2, whilst a more realistic model should also include an average in these frequencies. Obviously this cannot make the final result go to zero, instead of yielding [37], because [37] is already proportional to ω_1^2 and ω_2^2. Anyway such an average in the frequencies could make the coefficient in front of the right-hand side of [37] slightly different from its present value. We do not try such an average here because we do not know which function represents the frequencies of oscillation of the electrons in different atoms in different physical conditions. The same can be said of the amplitudes A_{1-} and A_{2-}. This indicates that [37] should be utilized as only an approximation, so that even when we know the exact values of β, γ, etc., the values of the frequencies and amplitudes yielding the gravitational force will be only known approximately. A more serious limitation of our model is that we are here trying to derive not only Newton's law of gravitation but also inertia as an electromagnetic effect (this appears from the sixth-order term in [37] when we follow a procedure like that given in ref. 54). At first sight this might appear quite reasonable because the proportionality between inertial and gravitational masses (or between weight and inertia, as some would call it), which is sometimes known as the equivalence principle, suggests a strong interconnection between gravitation and inertia. This is one of the strongest empirical evidences in support of Mach's principle, according to which, the inertia of any body is due to its gravitational interaction with the remainder of the universe (59, 60). So as we are trying to derive gravitation from an electromagnetic interaction, it would seem natural to try to do the same with inertia. But the problem is that even the smaller charges, considered in this work as the building blocks to derive gravitation, such as the electron and the proton, are known to have inertia (they have inertial mass as is evident from the fact that they have linear momentum and kinetic energy, we can apply Newton's second law to them, etc). This means that inertia is not only a property of neutral bodies, but belongs as well to the simplest charges known to us. To overcome this difficulty within the frame work of this paper it is necessary to assume that the proton and even the electron are conjugate particles composed of smaller ones, so that their inertia could be due to an electromagnetic interaction between these even smaller particles. The fact that the proton and the electron have spin gives some support to this idea because it is difficult to imagine how a structureless point particle could have such an intrinsic property as spin. However at this point we will not deal with this subject any further as it would go far beyond the scope of the present work. We restrict ourselves to pointing out this limitation, which shows to a certain extent the finite range of applicability of this model.

We would like to emphasize here that this is an initial exploratory model. As such it has limitations and is open to reasonable objections. But our goal in this whole work is to try to show the possibility of such an approach, namely, to derive gravitation as an electromagnetic effect and to produce the correct orders of magnitude. In this way we hope to cast some light upon possible routes that can yield the unification of the forces of nature.

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