Testing Mach's principle in electrodynamics

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Abstract: We analyze the consequences of Mach's principle as applied to electromagnetism. This view reveals new effects not predicted by standard electromagnetic theory.

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Résumé : Nous analysons les conséquences du principe de Mach appliqué à l'électromagnétisme. Ceci révèle de nouveaux effets non prévus par la théorie électromagnétique standard.

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1. Introduction

According to Newtonian mechanics there are motions of bodies relative to empty space and we can detect these motions when the bodies are accelerated (relative to absolute space, as Newton put it, or relative to inertial frames of reference, as we would say today). For instance, how can we know that the Earth really rotates around the north–south axis with a period of 1 day? The phenomenon of day and night does not prove the rotation of the Earth, as it can also be interpreted due to the diurnal translation of the Sun around a stationary Earth. According to Newton, however, the flattening of the Earth at its poles is a proof of its rotation. The reason is that, in his theory, no flattening would be created if the Earth were at rest in absolute space while the Sun and stars rotated around the north–south axis of the Earth with a period of 1 day. In Newton's theory, the amount of flattening is independent of the amount of matter in the surrounding astronomical bodies, so that if we could double the number of stars and galaxies, or make the Earth rotate alone in an otherwise empty Universe, the same amount of flattening should result. Foucault's pendulum gives other evidence for the real rotation of the Earth. Let us consider the simplest case of a pendulum swinging at the north pole of the Earth. The plane of oscillation does not remain fixed relative to the Earth but suffers a precession with a period of 1 day. In classical mechanics this is interpreted as due to the Earth's absolute rotation without any relation to the distant universe (stars and galaxies). If the Earth were at rest in absolute space and the remaining astronomical bodies were rotating as a whole around the north–south axis of the Earth with a period of 1 day, then the plane of oscillation of a swinging pendulum located at the north pole would not suffer any precession according to Newtonian theory, but would remain fixed relative to the surface of the Earth.

Leibniz and Berkeley (see ref. 1 Chap. 5) and Mach (see ref. 1 Chap. 6) rejected the concept of absolute space and proposed that there are only motions of bodies relative to other bodies. Accordingly, only these relative motions between material bodies could be detected or lead to measurable effects.

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This idea became known as Mach's principle. From this point of view the flattening of the Earth or the precession of Foucault's pendulum are due to the relative rotation between the Earth and the distant astronomical bodies. Accordingly, we cannot prove that the Earth really rotates, as the same phenomena should also happen in a stationary Earth while the distant astronomical bodies orbit around the north–south axis of the Earth with a period of 1 day. That is, if we could keep the Earth stationary and rotated the distant universe around the north–south axis of the Earth in the opposite direction with a period of 1 day, a pendulum swinging at the north pole should suffer a precession following the rotation of the Universe, the Earth should become flattened at its poles etc. This means that we cannot distinguish between the two situations, as both of them lead to the same observed effects. Moreover, if it were possible to annihilate the distant astronomical bodies (make their masses go to zero), the flattening of the Earth should also disappear, the same happening with the precession of a pendulum swinging at the north pole. These phenomena prove only the existence of a relative rotation between the Earth and the distant astronomical bodies, but not the rotation of the Earth itself relative to empty space. And if there was no relative rotation between the Earth and the distant astronomical bodies (or if the distant stars and galaxies could be annihilated), the pendulum should not suffer a precession, the shape of the Earth should become spherical etc. That is, if the joint rotation of the distant astronomical bodies relative to the Earth could be stopped, the flattening of the Earth or the precession of the swinging pendulum should also stop according to Mach's ideas.

Einstein coined the expression "Mach's principle" in 1918, see ref. 2 pp. 185–186 for an English translation of the relevant passage. He created his general theory of relativity in 1916 trying to implement Mach's ideas mathematically, [3]. In 1922, he presented one of the clearest formulations of Mach's principle, namely (see ref. 4 pp. 95–96),

> What is to be expected along the line of Mach's thought?
>
> 1. The inertia of a body must increase when ponderable masses are piled up in its neighbourhood.
> 2. A body must experience an accelerating force when neighbouring masses are accelerated, and, in fact, the force must be in the same direction as that acceleration.
> 3. A rotating hollow body must generate inside of itself a "Coriolis field", which deflects moving bodies in the sense of the rotation, and a radial centrifugal field as well.

The Coriolis field mentioned in the third point (which does not exist in Newtonian gravitational theory) would rotate the plane of oscillation of a pendulum oscillating at the north pole of a hypothetical stationary Earth due to the joint diurnal rotation of the surrounding stars and galaxies around the Earth. This effect was first derived in general relativity by Thirring and Lense in 1918 and 1921, see ref. 5 for an English translation of the relevant papers. In this work, we consider an electrodynamic analogy to this effect.

Since we cannot control the motion of the distant Universe nor the amount of matter it contains, our idea here is to explore the consequences of Mach's ideas in electromagnetism.

2. An electric Foucault's pendulum

There is an effect analogous to Foucault's experiment when we deal with classical electromagnetism. We will perform a thought experiment with a charged pendulum. Let us suppose that there is a pendulum of mass m and length ℓ oscillating in a vertical plane due to an uniform gravitational field g. The frequency of oscillation for small amplitudes is given by $\omega = \sqrt{g/\ell}$. If we are in an inertial frame of reference, the plane of oscillation of the pendulum will not suffer any precession. Now consider a charge q attached to the mass of the pendulum and place it in an uniform magnetic field $\boldsymbol{B}$ pointing upwards. The plane of oscillation will suffer a precession relative to the inertial frame of reference with an angular frequency given by $\Omega = -qB/2m$, supposing a weak magnetic field such that $|qB/m\omega| \ll 1$, see ref. 1 p. 45. The negative value of Ω indicates a rotation in the clockwise direction when the pendulum with a positive oscillating charge is seen from above.

There are three basic ways of creating an uniform magnetic field. These are the regions (*i*) near the poles of a large magnet, (*ii*) inside a long coil carrying a constant current (or equivalently the region near the center of Helmholtz's coils), or (*iii*) inside an uniformly charged spherical shell spinning with a constant angular velocity. To make the analogy with Foucault's experiment, we will consider the magnetic field due to the spinning of a charged spherical shell. Let us suppose that a spherical shell of radius R with uniformly distributed charge Q spins with a constant angular velocity $\boldsymbol{\omega}_Q$ relative to an inertial frame of reference. According to classical electromagnetism, ref. 6 p. 61, this system creates a dipole magnetic field outside the shell and a constant and uniform magnetic field anywhere inside the shell given by $\boldsymbol{B} = \mu_o Q\boldsymbol{\omega}_Q/6\pi R$, where $\mu_o = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of the vacuum. The precession of the plane of oscillation of the charged pendulum inside this shell relative to the inertial frame of reference will have the angular frequency $\Omega = -\mu_o q Q\omega_Q/(12\pi m R)$. When $qQ > 0\,(qQ < 0)$ then $\Omega\omega_Q < 0\,(\Omega\omega_Q > 0)$, indicating rotations in the opposite (the same) directions. From this expression, we can see that if $\omega_Q = 0$, then $\Omega = 0$. This would be analogous to stopping the rotation of the distant Universe in Foucault's experiment, leading in this case to no precession of the plane of oscillation. We can also see that if the amount of charge Q in the surrounding shell goes to zero, the same thing happens with the precession of the oscillating charged pendulum. This would be analogous to annihilating the distant astronomical bodies in Foucault's original experiment. This electric Foucault's pendulum lends support to the points of view expressed by Leibniz and Berkeley, and Mach.

3. Magnetic induction

For a more experimentally plausible case, let us replace the pendulum by a conductor ring of radius r placed at rest inside the spinning charged shell above, with the axis of the ring coinciding with the rotation axis of the shell and parallel to the uniform magnetic field $\boldsymbol{B}$ created by the shell. If this magnetic field is constant in time, no induction will happen in the ring. But if the magnetic field changes in time, there will be an electromotive force, emf, arising in the ring according to Faraday's law. It is given by $emf = -\mathrm{d}\Phi/\mathrm{d}t$, where $\Phi = \int\int \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{a} = B\pi r^2$ is the flux of the uniform magnetic field across the area of the ring, with $\mathrm{d}\boldsymbol{a}$ being an element of area. This electromotive force can be detected by the induced current generated in the ring. A change in the magnetic field can be accomplished by changing the rate of rotation of the shell, i.e., accelerating or decelerating it. The electromotive force in this case will then be given by: $emf = -\pi r^2\,\mathrm{d}B/\mathrm{d}t = -(\mu_o r^2 Q(\mathrm{d}\omega_Q/\mathrm{d}t))/6R$.

According to Mach's principle the same effect (emf and induced current in the ring) should happen if, instead of changing the rate of rotation of the shell relative to an inertial frame of reference, we change the rate of rotation of the internal ring with the opposite value. That is, if the charged spherical shell remains with a constant angular velocity relative to an inertial frame of reference and there is a change in the angular rotation of the internal ring given by $\mathrm{d}\omega_r/\mathrm{d}t$, there will be on it an induced emf given by: $emf = (\mu_o r^2 Q(\mathrm{d}\omega_r/\mathrm{d}t))/6R$. This effect is independent of the value of the constant angular rotation of the external shell, including the case of a stationary shell.

Conventional electromagnetic theory does not predict this effect. More precisely, it predicts the emf to be zero in this case, as is easily shown. Consider a charge q moving with velocity $\boldsymbol{v}$ relative to an inertial frame of reference, in the presence of electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively. In classical electromagnetism, the force $\boldsymbol{F}$ acting on the charge is given by Lorentz's force, namely, $\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$. A uniformly charged shell spinning with a constant angular velocity generates inside it only a uniform magnetic field parallel to the axis of rotation ($\boldsymbol{B} = \mu_o Q\boldsymbol{\omega}_Q/6\pi R$) and no electric field. Consider this axis of rotation to be the z axis and the ring of radius r spinning in the $z = 0$ plane along the poloidal direction, $\hat{\varphi}$. A conduction electron in the ring moving with tangential velocity $v = r\dot{\varphi}\hat{\varphi}$ will then suffer a magnetic force along the radial $\hat{r}$ direction, yielding no net *emf* along the ring. In the particular case of a stationary charged external shell, there are no fields at all inside it, showing directly that no *emf* nor induced current should happen in this case.

In principle the existence or not of this predicted *emf* (or induced current) can be tested in the laboratory. Other electromagnetic phenomena inside stationary charged shell are discussed elsewhere [7].

We are considering two cases here (*a*) changing the rate of rotation of the surrounding shell while the internal ring remains stationary or (*b*) changing the rate of rotation, in the opposite direction, of the internal ring while the surrounding shell remains stationary. These two cases are not exactly Mach-equivalent. Although the motion of the shell relative to the ring is the same in both cases, the motion of the ring relative to the rest of the Universe is not the same in cases (*a*) and (*b*). The reason is that in the first case the ring is not accelerated relative to the distant astronomical bodies, while in the second case there is such acceleration. However, we can disregard this asymmetry as the measurable effects due to it are usually very small [7].

Another way to change the magnetic flux without changing the rate of rotation of the spinning shell is to change the charge in the shell. For instance, by discharging the spinning shell the magnetic flux through the stationary ring rapidly decreases. An *emf* is induced in the ring according to Faraday's law.

Mach's principle implies that only the relative motion between the shell and the ring matters. Therefore, when the charged shell remains stationary and the ring rotates with the same constant angular velocity in the opposite direction (to generate the same relative motion between the ring and shell as in the previous paragraph), the same induction should occur in the ring when the shell is discharged. Induction should occur without an apparent magnetic field according to Mach's principle. Classical electromagnetism predicts no induction in this case. Once more, the existence or not of the effect should be tested in the laboratory.

4. Conclusion

This paper elucidated the crucial discrepancies between the predictions of conventional electromagnetic theory and those based on Mach's principle. The new effects described above test the validity of the implications of Mach's principle in electromagnetism. Only experiment can decide how Nature works.

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References

1. A.K.T. Assis. Relational mechanics. Apeiron, Montreal. 1999. ISBN: 0-9683689-2-1.
2. J.B. Barbour and H. Pfister (*Editors*). Mach's principle: from Newton's bucket to quantum gravity. Birkhäuser, Boston. 1995.
3. A. Einstein. The foundations of the general theory of relativity. *In* The principle of relativity. *Edited by* A. Einstein, H.A. Lorentz, H. Weyl, and H. Minkowski. New York, Dover. 1952. pp. 109-164.
4. A. Einstein. The meaning of relativity. Chapman and Hall, London. 1980.
5. B. Mashhoon, F.H. Hehl, and D.S. Theiss. On the gravitational effects of rotating masses: the Thirring-Lense papers. General relativity and gravitation. Vol. 16. Kluwer Academic, New York. 1984. pp. 711-750. ISSN: 0001-7701.
6. V.V. Batygin and I.N. Toptygin. Problems in electrodynamics. Academic Press, London. 1964.
7. A.K.T. Assis, J. Fukai, and H.B. Carvalho. Phys. Lett. **268A**, 274 (2000).