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ON MACH'S PRINCIPLE

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We propose the postulate that the resultant force acting on any body is zero. With this postulate and with a Weber force law for gravitation, we obtain equations of motion and conclude that all inertial forces are due to gravitational interaction with other bodies in the universe, as suggested by Mach. We then obtain the same value for the advance of the perhelion of the planets as is given by general relativity. All this is accomplished in a strictly relational theory. Finally, we relate these points to topical questions of electrodynamics raised by the experimental studies of Graneau and Pappas.

Key words: Mach's principle, Weber's law, equivalence principle.

1. INTRODUCTION

The point of view that kinematically equivalent motions must be dynamically equivalent is the most intuitive one. Despite its enormous success, Newtonian theory [1] does not satisfy this principle, because of the notion of absolute space and its influence on accelerated bodies (the famous bucket experiment). Historically, this led to criticism of Newtonian dynamics, in particular by Berkeley [2 - 4], Leibniz [5, 6], and Mach [7]. However, a completely relational theory capable of matching the amplitude and success of Newton's theory of motion has not hitherto been constructed. Einsten devised general relativity trying to incorporate Mach's idea that inertia is only due to gravitational interactions with the matter of the universe. He was only partially successful in this respect, as he himself emphasized [8–11], because he obtained a solution of his fields equations in which a single particle in an otherwise empty universe had inertial properties. It also seems that Mach did not consider himself as a forerunner of the relativists [12–14].

The goal of the present work is to give a relational theory for gravitation and from it to arrive at Mach's idea that inertial forces come from gravitational interactions of any body with the rest of the universe. A general framework for Machian theories, somewhat different from the present one, has been presented by Barbour and Bertotti [15].

This paper also presents a formal connection between gravitation and electromagnetism. In this aspect, the recent experimental results of Graneau [16–18] and Pappas [19] are of great importance. This is discussed in the final part of the present paper.

2. BASIC PRINCIPLES

The aim of this paper is to propose a physics which depends only on the relations between bodies and which is independent of the state of motion of the observer. We implement Mach's idea [7] that the inertial forces on any body are due to gravitational interactions between this body and other bodies in the universe. To this end, we introduce three postulates, valid for any kind of force (gravitational, electric, nuclear, etc.):

(A) Force is a vectorial quantity.

(B) The force that a material body A exerts on a material body B is equal and opposite to the force that B exerts on A.

(C) The sum of all forces on any material body is zero.

Postulate (A) qualifies in general the nature of a force. Postulate (B) is Newton's law of action and reaction. Postulate (C) replaces Newton's first and second laws, and also his first corollary [1], and can be called the principle of superposition of forces. A particular form of this postulate, valid only for gravitational forces, was given by Sciama [11]. This postulate and the results we will obtain with it are the main points of the present paper.

In order to implement these postulates and obtain the equations of motion on a relational basis [15, 20], we need some expressions for the forces. In this paper we are concerned with electromagnetic and gravitational forces, and for them we use a model in which the force that a material point j exerts on a material point i is given by

$$\vec{F}_{ji} \;=\; \kappa\,\frac{\hat{r}_{ij}}{r_{ij}^2}\Big[1+\frac{\xi}{c^2}\Big(r_{ij}\ddot{r}_{ij}-\frac{\dot{r}_{ij}^2}{2}\Big)\Big]$$

$$= \kappa \frac{\hat{r}_{ij}}{r_{ij}^2}\left[1 + \frac{\xi}{c^2}\left(\vec{v}_{ij}\cdot\vec{v}_{ij} - \frac{3}{2}(\hat{r}_{ij}\cdot\vec{v}_{ij})^2 + \vec{r}_{ij}\cdot\vec{a}_{ij}\right)\right], \qquad (1)$$

where

$$\begin{cases} \vec{r}_{ij} \equiv \vec{r}_i - \vec{r}_j, \quad r_{ij} \equiv |\vec{r}_i - \vec{r}_j|, \\ \hat{r}_{ij} \equiv \dfrac{\vec{r}_{ij}}{r_{ij}}, \\ \dot{r}_{ij} \equiv \dfrac{d}{dt}\, r_{ij}, \quad \ddot{r}_{ij} \equiv \dfrac{d^2}{dt^2}\, r_{ij}, \\ \vec{v}_{ij} \equiv \vec{v}_i - \vec{v}_j, \quad \vec{a}_{ij} \equiv \vec{a}_i - \vec{a}_j, \\ c \equiv \text{light velocity} = (\varepsilon_0\mu_0)^{-1/2} = 2.998 \times 10^8\, ms^{-1}. \end{cases} \qquad (2)$$

In the case of electromagnetism, this is equivalent to Weber's law [21, 22] with

$$\kappa = H_e q_i q_j, \quad \xi = 1, \qquad (3)$$

where WE ~~the~~ assume that H_e is a constant.

In the case of gravitation, we propose a modification of Newton's law of gravitation (which is Eq.(1) with $\kappa = -Gm_i m_j$ and $\xi = 0$), the new law being

$$\kappa = -H_g m_i m_j, \quad \xi = 6, \qquad (4)$$

where we assume that H_g is a constant. Later we will see why $\xi = 6$. A law of this kind was first proposed by Tisserand [23, 24].

Equation (1) has the following important properties:

(A) It depends only on the relative distance, velocity, and acceleration of the two particles, and so it is completely relational in nature. Thus, it has the same value for any observer, irrespective of whether or not the observer is inertial from the Newtonian point of view.

(B) It satisfies the second postulate strictly.

(C) The resulting equation of motion is obtained by the third postulate in conjunction with Eq.(1).

(D) The force (1) can be derived from a velocity–dependent potential energy [22, 25]:

$$U = \frac{\kappa}{r_{ij}}\left(1 - \frac{\xi}{2c^2}\,\dot{r}_{ij}^2\right), \qquad (5)$$

from which the force (1) is obtained by differentiating this energy with respect to r_{ij} and then reversing the sign. From the postulates and from (5) the conservation of energy also follows.

It should be pointed out that the third postulate, applied to forces (1) to (4), has not hitherto been proposed. This is the principal contribution of the present work.

3. MACH'S PRINCIPLE

We now show how to develop Mach's ideas on the basis of these postulates. From Eq.(1) we find that a spherical shell of radius r, thickness dr, with an isotropic mass distribution $\rho(r)$ around its center, and spinning with angular velocity $\vec{\omega}(t)$, attracts a material point m_1 localized outside the spherical shell with a force given by

$$\begin{aligned} d\vec{F} = \frac{-H_g m_1[4\pi r^2\rho(r)dr]}{r_1^2} \Bigg\{\Bigg[1 + \frac{\xi}{c^2}\Bigg(v_1^2 - \frac{3}{2}(\hat{r}_1\cdot\vec{v}_1)^2 \\ + \ \vec{r}_1\cdot\vec{a}_1\Bigg)\Bigg]\hat{r}_1 + \ \frac{\xi}{c^2}\frac{r^2}{r_1^2}\Bigg[\frac{r_1}{3}\,\vec{a}_1 - (\hat{r}_1\cdot\vec{v}_1)\vec{v}_1 - \frac{v_1^2}{2}\,\hat{r}_1 \\ + \ \frac{5}{2}\,(\hat{r}_1\cdot\vec{v}_1)^2\hat{r}_1 - (\vec{r}_1\cdot\vec{a}_1)\hat{r}_1 + (\hat{r}_1\cdot\vec{v}_1)(\vec{\omega}\times\vec{r}_1) \\ + \ \frac{2}{3}\,r_1(\vec{v}_1\times\vec{\omega}) + \frac{r_1}{3}(\vec{\omega}\cdot\vec{r}_1)\vec{\omega} + \frac{r_1^2\omega^2}{6}\,\hat{r}_1 - \frac{(\vec{r}_1\cdot\vec{\omega})^2}{2}\,\hat{r}_1 \\ + \ [\vec{r}_1\cdot(\vec{\omega}\times\vec{v}_1)]\hat{r}_1 + \frac{r_1}{3}\Bigg(\vec{r}_1\times\frac{d\vec{\omega}}{dt}\Bigg)\Bigg]\Bigg\}. \end{aligned} \tag{6}$$

If m_1 were localized inside the shell, the force would be given by

$$\begin{aligned} d\vec{F} = \frac{-4\pi}{3}\,H_g\frac{\xi}{c^2}m_1\rho(r)rdr\Bigg[\vec{a}_1 + \vec{r}_1\times\frac{d\vec{\omega}}{dt} + 2\vec{v}_1\times\vec{\omega} \\ + \ \vec{\omega}\times(\vec{\omega}\times\vec{r}_1)\Bigg], \end{aligned} \tag{7}$$

where in these equations $\vec{r}_1$, $\vec{v}_1$, and $\vec{a}_1$ are, respectively, the radius vector, velocity, and acceleration of body 1 relative to the centre of the spherical shell.

In order to obtain the equations of motion, we need to include, in accordance with the third postulate, the interactions between all the bodies in the universe. We can divide the forces acting on a body m_1 into two parts. The first part is its interaction with local bodies and with anisotropic distributions of bodies surrounding it. The second part is its interaction with isotropic distributions of bodies which surround it. We first calculate the contribution of this last part. It is a known fact that the universe is remarkably isotropic when measured by the integrated microwave and X-ray backgrounds, or by radio source counts and deep galaxy counts [26–32]. As the earth does not occupy a central position with respect to the universe, this fact suggests homogeneity on a very large scale. Due to the great distance between the stars and to their charge neutrality, the stars can only interact significantly with any distant body by gravitational forces. From (7) we find that the force on m_1 due to the isotropic distribution of stars and galaxies is given by

$$\vec{F} = -\Phi m_1[\vec{a}_1 + \vec{r}_1 \times \frac{d\vec{\omega}}{dt} + 2\vec{v}_1 \times \vec{\omega} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_1)], \qquad (8)$$

where

$$\Phi \equiv \frac{4\pi}{3} H_g \frac{\xi}{c^2} \int_0^R \rho(r) r\, dr \qquad (9)$$

and where R is the radius of the observable universe at the present epoch. The Hubble constant H_0 is related to R by $R = c/H_0$. With the hypothesis of homogeneity, we obtain

$$\Phi = \frac{2\pi}{3} H_g \frac{\xi}{c^2} \rho_0 R^2 = \frac{H_g}{2} \frac{\xi}{c^2} \frac{M}{R}, \qquad (10)$$

where M is the mass of the observable universe.

Let us represent the force on m_1 due to its interaction with local bodies and with anisotropic distributions of mass surrounding it by

$$\vec{F} = \sum_{j=2}^{N} \vec{F}_{j1}, \qquad (11)$$

where $\vec{F}_{j1}$ is the force that the body j exerts on m_1. The equation of motion for m_1 is then obtained by adding forces (8) and (11) and using the third postulate to equate the result to zero:

$$\sum_{j=2}^{N} \vec{F}_{j1} - \Phi m_1[\vec{a}_1 + \vec{r}_1 \times \frac{d\vec{\omega}}{dt} + 2\vec{v}_1 \times \vec{\omega}$$
$$+ \ \vec{\omega} \times (\vec{\omega} \times \vec{r}_1)] = 0. \qquad (12)$$

If we are in a frame of reference in which the "fixed stars" (i.e., the distant bodies of the universe as, for instance, the most distant galaxies) are not rotating, then the equation of motion will be given by the simple expression

$$\sum_{j=2}^{N} \frac{\vec{F}_{j1}}{\Phi} = m_1\vec{a}_1, \tag{13}$$

which is similar to Newton's second law. A particular case of Eq.(13) is when m_1 is interacting only with isotropic distributions of bodies surrounding it. This situation would be the one similar with Newton's first law. From Eqs. (12) and (13) we see that the inertial frames, i.e., those for which Newton's laws of motion hold without the introduction of Coriolis or centrifugal forces, are the ones which are unaccelerated relative to the "fixed stars." The model presented in this paper thus gives a causal law for the observational fact [33] that if there is a relative rotation between the two frames (a inertial frame and the frame defined by the "fixed stars") it is smaller than 2×10^{-8} rad/yr. Observing the large scale isotropy of the X-ray and microwave background, we obtain more stringent limits on the vorticity of the universe relative to absolute space [34–38]. In classical physics, it is a coincidence that the inertial frames are those which are nonrotating relative to the distant stars and galaxies. The same can be said of general relativity, because such a result is only true for a special class of cosmological models [35, 36]. The second important point to note about the above equation is that it explains the proportionality between the gravitational and inertial masses. The reason is that the force on (8) is in fact a gravitational interaction between any body and the isotropic distribution of stars and galaxies surrounding it. An important consequence of this theory is that G, the Newtonian gravitational "constant," should be a function of the time, due to the expansion of the universe. To see this, suppose that the body of mass m_1 interacts only with an uncharged body of mass m_2 and with the isotropic distribution of stars. From (1) and (13) we then find, when $r_{12}\ddot{r}_{12} << c^2$ and $\dot{r}_{12}^2 << c^2$,

$$\frac{-H_g}{\Phi}\, m_1 m_2\, \frac{\hat{r}_{12}}{r_{12}^2} = m_1\vec{a}_1. \tag{14}$$

From (14) and (9) or (10) we obtain

$$G = \frac{2}{\xi}\,\frac{Rc^2}{M} = \frac{3c^2}{2\pi\xi\rho_0 R^2} = \frac{3H_0^2}{2\pi\xi\rho_0}. \tag{15}$$

Because R/M varies due to the expansion of the universe, we conclude that G varies with time. An alternative interpretation of this equation is that G is a constant but ρ_0 is a function of time. Astronomical evidence for variation of G with time was obtained by Van Flandern [39], although this is still a controversial result [40, 41]. An expression like (15) was obtained by Sciama [11] and also by Dicke [42], but without a specific constant, whereas ξ in (15) is given by (4). The relation (15) is a necessary consequence of the model presented in this paper. It is known to be approximately valid, but the value of R/M or $\rho_0 R^2$ is not yet accurately known, due to uncertainties in the measures of the mean density of the universe and of the Hubble constant [43–45]. From Eq.(14) we can see, observing that Φ is proportional to H_g, that from this equation we can not calculate or estimate H_g, because it cancels out of the expression.

If we have two charges q_1 and q_2 interacting with one another and with the isotropic distribution of stars, we would have from (1), (3), and (13) that $H_e/\Phi = (4\pi\varepsilon_0)^{-1}$. So, although we cannot estimate the value of H_g, we can obtain the ratio $H_e/H_g = (\xi\rho_0 R^2)/(6\varepsilon_0 c^2) = (\xi\rho_0)/(6\varepsilon_0 H_0^2)$. If ρ_0 is a constant, then we conclude that not only G but also ε_0 should be a function of time. The other possibility is to have G and ε_0 as constants, and then ρ_0 will be a function of time, so that $\rho_0/H_0^2 =$ constant for any t.

It should be emphasized here that all these conclusions of the temporal variation of H_0, G, ρ_0, etc., arose from the assumption that the Hubble law of red shifts ($\Delta\lambda/\lambda_0 \simeq rH_0/c$) is due to a Doppler effect arising from the recession between galaxies. The idea that the universe is expanding comes from this interpretation of the redshifts. If it is found that the physical mechanism responsible for the Hubble law is a different one, then the previous discussion should be redone.

We now discuss the question of anisotropy of the inertial mass. It has been suggested in the literature, especially by Cocconi and Salpeter [46], that an anisotropic distribution of matter surrounding a body would cause, according to their interpretation of Mach's principle, an anisotropy in the inertial mass of this body. Accordingly, the earth, the sun, our Galaxy, or the Virgo Supercluster would cause an anisotropy in the inertial mass of any body on the surface of the earth. This conclusion resulted from the assumption that the inertial mass of any body arises from its gravitational interaction with *all* other bodies in the universe. Then a body on the surface of the earth would have an inertial mass in the direction of the sun with a different value from the inertial mass perpendicular to it. In other words, the inertial mass of any body would be a tensor of order 2. This was refuted by experiments, especially those of Hughes et al. [47], and Drever [48]. These gave an upper for the mass anisotropy of 5×10^{-23}, which was many orders of magnitude smaller than the values predicted by Cocconi and Salpeter.

The model presented in this paper offers a simple explanation for these results. The important point to note is that the only bodies which contribute to the inertial mass of any mass m_1 are the isotropic distributions of matter around it. These are the bodies which give a zero resultant force according to the Newtonian law of gravitation (to see this, we simply put $\xi = 0$ in (8) and (9)). But, in accordance with a Weber law for gravitation, these bodies will give a force proportional to the acceleration and the mass of the body inside. All the local bodies and anisotropic distributions of matter around a mass m_1 will have their total contribution included on the left-hand side of (13), as in usual classical mechanics. Thus, the inertial masses which arise in this model should in fact be scalars. We can state concisely Mach's principle as we see it in the following form: The inertial mass of a body is caused by its interaction with the isotropic matter distributions arount it.

We can see from (8) that all inertial forces (Coriolis, centrifugal, etc.) are in this model real gravitational forces, produced by the relative accelaration between the body which experiences these forces and the "fixed stars." With a Weber law for the gravitational force, Mach's ideas are fully implemented in a completely relational way. We hope that this can cast some light on this whole subject, because the lack of a positive theory was always a problem for the acceptance of Mach's points of view [49].

As a last point, we now show how to obtain the kinetic energy in this formulation. The gravitational potential energy of a material point m_1 inside a nonrotating spherical shell of radius r, thickness dr, and with an isotropic mass distribution $\rho(r)$ is given as a result of integrating Eq.(5) and is

$$dU = -4\pi H_g m_1 \rho(r) r dr \left(1 - \frac{\xi}{6} \frac{v_1^2}{c^2}\right), \tag{16}$$

where $\vec{v}_1$ is the velocity of the material point m_1 relative to the center of the spherical shell. Integrating this quantity for the isotropic distribution of stars, one gets

$$U = \Phi\left(\frac{m_1 v_1^2}{2} - \frac{3}{\xi} m_1 c^2\right), \tag{17}$$

where ξ is given by (4) and Φ by (9) or (10). From (17) we observe that the kinetic energy and half the rest energy of any particle are due to gravitational interaction with the isotropic distribution of stars and galaxies around it.

4. PRECESSION OF THE PERIHELION

We now use Eq.(1) to calculate the orbit of a planet around the sun. With sufficient accuracy we can assume that the planets are material points, because their diameters are much smaller then their distance to the sun. We show that the sun can also be assumed to be a material point. In the solar system, the maximal value of r^2/r_1^2 in (6) is $R_s^2/d_M^2 \sim 1.4 \times 10^{-4}$, where R_s is the radius of the sun and d_M is the distance between the planet Mercury and the sun. With this maximum value, together with the value of the angular rotation of the sun around its axis, $\omega = 2.9 \times 10^{-6} s^{-1}$, and the values of the translational velocity and distance to the sun for Mercury [50], we find that all terms which multiply the second ξ in (6) are at least 5×10^{-4} times smaller than the values that multiply the first ξ in (6). At the present time, there is still no consensus about the value of the angular rotation of the solar core, but an usual estimate is that it is 2 to 9 times greater than the surface rotation [51, 52]. Utilizing these data for $\omega(r)$, we find that the terms in the second ξ will be at least 6×10^{-4} times smaller than those in the first ξ. Then we can in a first approximation neglect all terms of the second ξ and treat the sun as a material point. It is easy to see that in this limit, namely, $r/r_1 \to 0$, Eq.(6) simplifies to Eq.(1).

From Eqs.(1), (10), (13), and (15) we obtain the equations of motion for a planet and for the sun when they are interacting gravitationally with one another and with the "fixed stars" (using the fact that $\hat{r}_{sp} = -\hat{r}_{ps}$):

$$m_s \vec{a}_s = \frac{-Gm_s m_p}{r_{sp}^2} \hat{r}_{sp} \left[1 + \frac{\xi}{c^2}\left(r_{sp}\ddot{r}_{sp} - \frac{\dot{r}_{sp}^2}{2}\right)\right], \tag{18}$$

$$m_p \vec{a}_p = \frac{+Gm_s m_p}{r_{sp}^2} \hat{r}_{sp} \left[1 + \frac{\xi}{c^2}\left(r_{sp}\ddot{r}_{sp} - \frac{\dot{r}_{sp}^2}{2}\right)\right]. \tag{19}$$

Adding these two equations, we obtain the conservation of the total linear momentum:

$$m_s \vec{a}_s + m_p \vec{a}_p = \frac{d}{dt}(m_s \vec{v}_s + m_p \vec{v}_p) = 0. \tag{20}$$

If we subtract Eq.(19) from (18), we obtain

$$\vec{a}_{sp} \equiv \vec{a}_s - \vec{a}_p = \frac{-G(m_s + m_p)}{r_{sp}^2} \hat{r}_{sp} \left[1 + \frac{\xi}{c^2}\left(r_{sp}\ddot{r}_{sp} - \frac{\dot{r}_{sp}^2}{2}\right)\right], \tag{21}$$

which shows that $\vec{a}_{sp}$ is parallel to $\hat{r}_{sp}$. From this we obtain conservation of the angular momentum, which is defined by

$$\vec{L}_{sp} \equiv \vec{r}_{sp} \times (M\vec{v}_{sp}), \tag{22}$$

where $M \equiv m_s + m_p$ is the total mass and $\vec{v}_{sp} \equiv \vec{v}_s - \vec{v}_p$. This means that $\vec{r}_{sp}$ always lies in a plane whose normal is parallel to the constant

$\vec{L}_{sp}$, as in Newtonian mechanics. If we choose the coordinate system with the z axis parallel to $\vec{L}_{sp}$, the motion of the system will then always be in the xy plane. Writing Eq.(21) in plane polar coordinates centred on the sun, we obtain two equations, one for the $\hat{\varphi}$ component of the equation and one for the $\hat{\rho}$ component, namely,

$$\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi} = 0, \tag{23}$$

$$\ddot{\rho} - \rho\dot{\varphi}^2 = -GM\left[\frac{1}{\rho^2} + \frac{\xi}{c^2}\left(\frac{-\dot{\rho}^2}{2\rho^2} + \frac{\ddot{\rho}}{\rho}\right)\right]. \tag{24}$$

Equation (23) is the conservation of angular momentum. This implies that the quantity

$$H \equiv \rho^2\dot{\varphi}, \tag{25}$$

is constant at all t.

Up to now, the results are the same as in Newtonian mechanics. A difference appears only in Eq.(24) with the terms that multiply ξ. Defining $u \equiv 1/\rho$ and using the standard prescription [53], we obtain the following orbit equation from Eq.(24):

$$\frac{d^2u}{d\varphi^2} + u = GM\left[\frac{1}{H^2} - \frac{\xi}{c^2}\left(\frac{1}{2}(\frac{du}{d\varphi})^2 + u\frac{d^2u}{d\varphi^2}\right)\right]. \tag{26}$$

This equation can be solved iteratively by observing that the second and third terms in the square bracket are much smaller then the first one. We therefore seek a solution in the form

$$u(\varphi) = u_0(\varphi) + u_1(\varphi), \tag{27}$$

with $|u_0| >> |u_1|$, and where $u_0(\varphi)$ and $u_1(\varphi)$ satisfy the equations

$$\frac{d^2u_0}{d\varphi^2} + u_0 = \frac{GM}{H^2}, \tag{28}$$

$$\frac{d^2u_1}{d\varphi^2} + u_1 = -GM\frac{\xi}{c^2}\left[\frac{1}{2}(\frac{du_0}{d\varphi})^2 + u_0\,\frac{d^2u_0}{d\varphi^2}\right]. \tag{29}$$

The solution of Eq.(28) is the classical result

$$u_0(\varphi) = \frac{GM}{H^2} + A\,\cos(\varphi - \varphi_0), \tag{30}$$

where A and φ_0 come from the initial conditions. Using Eq.(30) in Eq.(29), we can obtain a particular solution for $u_1(\varphi)$:

$$u_1(\varphi) = a_1 + a_2(\varphi - \varphi_0)\sin(\varphi - \varphi_0) + a_3\,\cos^2(\varphi - \varphi_0), \tag{31}$$

where

$$\begin{cases} a_1 = \dfrac{GMA^2}{2}\ \dfrac{\xi}{c^2}, \\ a_2 = \dfrac{G^2M^2A}{2H^2}\ \dfrac{\xi}{c^2}, \\ a_3 = \dfrac{-GMA^2}{2}\ \dfrac{\xi}{c^2}. \end{cases} \tag{32}$$

The turning points, at which the distance of the planet to the sun is a maximun or a minimum, are given by $du/d\varphi = 0$. We can see from (27) to (32) that $\varphi = \varphi_0$ is one solution. After one revolution, the turning point will be near $\varphi_0 + 2\pi$. Expanding $du/d\varphi$ around this value and equating to zero, we obtain

$$\varphi \simeq \varphi_0 + 2\pi + \frac{2\pi a_2}{A}. \tag{33}$$

The advance of the perihelion in one revolution is then given by

$$\Delta\varphi = \pi\ \frac{\xi}{c^2}\ \frac{G^2M^2}{H^2} = \pi\ \frac{\xi}{c^2}\ \frac{GM}{a(1-\varepsilon^2)}, \tag{34}$$

where a is the semimajor axis and ε is the eccentricity of the orbit. With the value of ξ given by (4), we arrive at exactly the same result as the one obtained with general relativity [54], which is well verified experimentally. The calculation presented in this section was important to show in what points a Weber law for gravitation differs from Newton's law (that is, in all the terms in which ξ appears) and to exhibit a specific value of the constant ξ which fits the experimental results well.

It should also be mentioned that, although this model gives an identical result for the advance of the perihelion as that obtained by using general relativity, these theories are based on different concepts and mathematical tools. As examples, we mention the utilization in general relativity of Riemannian geometry, the strong equivalence principle, the Minkowski metric, and the Schwarzschild line element, etc [54, 55]. With the application of these aspects, the orbit equation obtained in general relativity is

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{H^2} + \frac{3GM}{c^2}\ u^2. \tag{35}$$

Comparison of this equation with (26) shows that they are not equivalent in general, although the solution is the same to first order, as we have shown. Differences apppear in the terms of second-order, and with the improvement of observations and experimental techniques this will be a good test for both theories in the future. Although it is hopeless nowadays to test second–order terms using the planet Mercury, due to observational errors for its precession, perhaps it will be possible to test these terms using a binary pulsar.

5. WEBER'S LAW BETWEEN ELECTRIC CHARGES

We now turn our attention to the Weber law between electric charges, which is Eqs.(1) to (3) with $(4\pi\varepsilon_0)^{-1}$ in place of H_e, and what can be done with it. This is relevant nowadays on account of the experimental researches of Graneau [16–18] and Pappas [19], and the connections between these facts are shown here.

It is known [22, 25] that with Weber's force we can obtain Ampère's law of force between current elements, which was based on a series of famous experiments and on the assumption that the force between current elements follows the law of action and reaction in the strongest form. To deduce Ampère's law from Weber's force, we proceed as follows: The force that a usual current element $I_2 d\vec{\ell}_2$ in wires or magnets exerts on another usual current element $I_1 d\vec{\ell}_1$ is given by the sum of four terms. These are the forces that the positive and negative charges of $I_2 d\vec{\ell}_2$ exert on the positive and negative charges of $I_1 d\vec{\ell}_1$. With this we obtain, from Eqs.(1) to (3),

$$d\vec{F}_{21} = \frac{-\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}_{12}}{r_{12}^2}\Big[2d\vec{\ell}_1 \cdot d\vec{\ell}_2 - 3(\hat{r}_{12}\cdot d\vec{\ell}_1)(\hat{r}_{12}\cdot d\vec{\ell}_2)\Big], \tag{36}$$

where the following relations have been used:

$$\begin{cases} \lambda_{i\pm} d\ell_i = q_{i\pm}, \quad i = 1 \qquad \text{or} \quad i = 2, \\ \lambda_{i-} = -\lambda_{i+}, \\ I_i d\vec{\ell}_i = \lambda_{i+} d\ell_i (\vec{v}_{i+} - \vec{v}_{i-}). \end{cases} \tag{37}$$

Nowadays, Eq.(36), Ampère's law of force between current elements, is little used, although Maxwell called it the cardinal formula of electrodynamics [56]. In almost all the textbooks we find only Grassmann's law [57], which is given by

$$\begin{aligned} d\vec{F}_{21} &= \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\, d\vec{\ell}_1 \times (d\vec{\ell}_2 \times \hat{r}_{12}) \\ &= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\Big[(d\vec{\ell}_1\cdot d\vec{\ell}_2)\hat{r}_{12} - (d\vec{\ell}_1 \cdot \hat{r}_{12})d\vec{\ell}_2\Big]. \end{aligned} \tag{38}$$

This law is sometimes called Biot–Savart's law, but in fact Biot and Savart gave only an expression for the magnetic field due to a current element, namely, $d\vec{B}_2(\vec{r}_1) = \mu_0 I_2(d\vec{\ell}_2 \times \hat{r}_{12})/(4\pi r_{12}^2)$. They arrived at this expression while studying the influence of an electric current in a magnet, [58].

Grassmann force does not satisfy in general Newton's law of action and reaction, whereas Eq.(36) does satisfy it, and this is one of the criticisms that is usually made against Eq.(38). On the other hand, Eqs.(36) and (38) give the same value of the force on $I_1 d\vec{\ell}_1$ when they are integrated over the entire circuit 2, if $I_1 d\vec{\ell}_1$ is not a part of circuit 2. The proof of this fact can be found in the book of Tricker [59]. Thus, when we are dealing with the force between two different circuits, we can still use Grassmann's law, which is much easier to integrate than Ampère's law. However, when $d\vec{\ell}_1$ is part of the closed circuit 2 the proof no longer holds, and the two laws are not equivalent. In particular, Ampère's law foresees longitudinal forces, while Grassmann's law does not, and in recent years a great number of experiments has been done with a single circuit which confirm Ampère's law. These experiments deal with jet propulsion in liquids [60]; railgun accelerators [16, 17, 61–63]; the exploding wire phenomena [64–69]; electrodynamic explosions in liquids [70–72]; and electromagnetic impulse pendulum [18, 19, 73, 74]. Moreover, it has been shown [75] that only the Ampère force law is compatible with the virtual–work concept, while the Grassmann force law is not in general in agreement with this concept.

Another important aspect of Weber's law is that with it we obtain Faraday's law of induction of electric current when the sources that induce the effect are closed loops [22, 25].

All these aspects show the importance of Weber's law and the importance of increasing theoretical and experimental research in this direction.

It is worth emphasizing here that with laws (1) to (4) we have a formal connection between electromagnetism and gravitation, in which the two forces present the same structure in this model. This indicates that phenomena similar to magnetism should occur in gravitation, although their magnitude may be much smaller due to the weakness of the gravitational constant. The experiments of Graneau and Pappas favour Ampère's law against Grassmann's law. As Weber's law produces Ampère's as a special case, this research permits the utilization of Weber force as a possible explanation for these phenomena.

6. DISCUSSION AND CONCLUSIONS

The main point of this paper was the introduction of the postulate which asserts that the sum of all forces of any kind acting on a body is zero, together with the use of a Weber force law for electric and gravitational interactions. In this model we have found that all inertial forces are in fact gravitational forces due to the interactions of any body with the isotropic distribution of matter around it. As this is a relational theory, Mach's ideas have been implemented and the role of inertial frames of reference have been clarified and identified with frames which are nonaccelerated relative to the "fixed stars."

The greatest limitation of this model is that it is based on an action–at–a–distance theory (see Eq.(1)). As a result, it is not a definitive or final theory but should be valid in systems with slowly varying motions in which time retardation is not a serious factor. A theory which involves the generation and propagation of gravitational waves will be presented in a future paper, but it can be mentioned here that the structure of Eqs.(1) to (4) strongly suggests that the velocity of gravitational waves will be the velocity of light or will be equal to $c/(\xi)^{\frac{1}{2}}$. These equations also suggest that the generation, propagation, and detection of gravitational waves should be similar to the case of electromagnetic waves, though it should be noted that in general we only have gravitational masses of the same kind, whereas in electromagnetism we readily obtain positive and negative charges. So, there should be no dipole radiation of gravitational waves. A very interesting proposal to extend Weber's law to include electromagnetic radiation, through the introduction of time retardation, was made recently by Wesley [25]. An earlier proposal in this direction was made by Parry Moon and Spencer [76]. An alternative way of obtaining time delays in an action–at–a–distance theory was given in Ref.[77]. The main idea is to obtain time delays by many–body interactions via a law of induction. All these ideas are important and should be further investigated. For a review of possible sources and methods of detection of gravitational waves see Ref.[78].

An important topic which we want to treat in a future work is the interaction of electromagnetic waves with matter. In particular, we want to deal with the deflection of light in a gravitational field and with the gravitational redshift. Due to the relevance of these topics we will treat them in a separate work.

In this paper, we have also shown how one can obtain exactly the same value for the advance of the perihelion as given by general relativity but on the basis of quite different concepts. Moreover, we have obtained an expression for the gravitational "constant" G which is dependent on the distribution of mass in the universe and on time. As a consequence, we disagree with Dirac's position [79], according to which a theory that does not satify the strong equivalence principle cannot explain the advance of the perihelion of Mercury. This shows the connection of this work to deep questions of cosmology.

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REFERENCES

1. I. Newton, *Principia* (University of California Press, Cajori edition, 1960).
2. G. Berkeley, *A Treatise Concerning the Principles of Human Knowledge*, Sec. 110-117 (1710); De Motu, Sec. 52-65 (1721); *The Works of George Berkeley* (Clarendon, Oxford, 1901).
3. K. R. Popper, *Br. J. Phil. Sci.* **4**, 26 (1953).
4. J. Myhill, *Univ. Calif. Publ. Philos.* **29**, 141 (1957).
5. *The Leibniz - Clark Correspondence,* H. G. Alexander, ed. (Manchester University Press, Manchester, 1956).
6. M. Bunge, *Am. J. Phys.* **34**, 586 (1966).
7. E. Mach, *The Science of Mechanics* (Open Court, La Salle, 1960).
8. A. Einstein, *S. B. Preuss, Akad. Wiss*, 1917, p. 142.
9. A. Einstein, *The Meaning of Relativity* (Chapman & Hall, London, 1967), p. 94 ff.
10. H. Bondi, *Assumption and Myth in Physical Theory* (Cambridge University Press, Cambridge, 1967), p. 74.
11. D. W. Sciama, *Mon. Not. R. Astron. Soc.* **113**, 34 (1953).
12. E. Mach, *The Principles of Physical Optics* (Methuen, London, 1926), Preface.
13. J. T. Blackmore, *Ernst Mach - His Life, Work and Influence* (University of California Press, Berkeley, 1972), pp. 257 and 282.
14. A. Pais, *'Subtle is the Lord ⋯' - The Science and the Life of Albert Einstein* (Oxford University Press, Oxford, 1982), p. 283.
15. J. B. Barbour and B. Bertotti, *Nuovo Cimento B* **38**, 1 (1977).
16. P. Graneau, *Fortschr. Phys.* **34**, 457 (1986).
17. P. Graneau, *Ampère-Neumann Electrodynamics of Metals* (Hadronic Press, Nonantum, Massachusetts, 1985).
18. P. Graneau and P. N. Graneau, *Nuovo Cimento D* **7**, 31 (1986).
19. P. T. Pappas, *Nuovo Cimento B* **76** (2), 189 (1983).
20. J. B. Barbour and B. Bertotti, *Proc. R. Soc. London, Ser. A* **382**, 295 (1982).
21. W. Weber, *Abh. Leibnizens Ges.* (Leipzig) p. 316 (1846); English translation in: *Taylor's Scientific Memoirs* V, p. 489 (1852); *Wilhekm Weber's Werke, Vols. 1-6* (Springer, Berlin, 1893).

l

22. J. C. Maxwell, *A Treatise on Electricity and Magnetism, Vol. 2,* (Dover, New York, 1954), Chap. XXIII.

23. M. F. Tisserand, C.R. Acad. Sci. (Paris) **75**, 760 (1872); C.R. Acad. Sci. (Paris) **111**, 313 (1890).

24. E. T. Whittaker, *A History of the Theories of Aether and Electricity - The Classical Theories* (Humanities Press, New York, 1973), pp. 207-208.

25. J. P. Wesley, *Spec. Sci. Technol.* **10** (1), 47 (1987).

26. R. B. Partridge, *Rep. Prog. Phys.* **51**, 647 (1988).

27. M. Davis and P. J. E. Peebles, *Ann. Rev. Astron. Astrophys.* **21**, 109 (1983).

28. H. Gursky and D. A. Schwartz, *Ann. Rev. Astron. Astrophys.* **15**, 541 (1977).

29. A. Webster, *Mon. Not. R. Astron. Soc.* **175**, 61 (1976); *ibid.*, p. 71.

30. P. J. E. Peebles, *The Large-Scale Structure of the Universe* (Princeton University Press, Princeton, 1980).

31. D. W. Sciama, *Modern Cosmology* (Cambridge University Press, Cambridge, 1973), Chap. 16.

32. D. J. Raine, *The Isotropic Universe* (Adam Hilger, Bristol, 1981).

33. L. I. Schiff, *Rev. Mod. Phys.* **36**, 510 (1964).

34. A. M. Wolfe, *Astrophys. J.* **159**, L 61 (1970).

35. D. J. Raine, *Rep. Prog. Phys.* **44**, 1151 (1981).

36. M. Reinhardt, *Z. Naturforsch.* **28A**, 529 (1973).

37. S. W. Hawking, *Mon. Not. R. Astron. Soc.* **142**, 129 (1969).

38. D. W. Sciama, in *The Physicist's Conception of Nature,* J. Mehra, ed. (Reidel, Dordrecht, 1973), p. 17.

39. T. C. van Flandern, *Mon. Not. R. Astron. Soc.* **170**, 333 (1975).

40. T. Damour, G. W. Gibbons, and J. H. Taylor, *Phys. Rev. Lett.* **61**, 1151 (1988).

41. R. D. Reasenberg and I. I. Shapiro, in *On the Measurements of Cosmological Variations of the Gravitational Constant,* L. Halpern, ed. (University Press of Florida, Gainesville, 1978), p. 71.

42. R. H. Dicke, *Phys. Rev. Lett.* **7**, 359 (1961); C. Brans and R. H. Dicke, *Phys. Rev.* **124** (3), 925 (1961); R. H. Dicke, *The Theoretical Significance of Experimental Relativity* (Blackie & Son, London, 1964).

43. R. B. Tully, *Nature* **334**, 209 (1988).

44. M. Rowan-Robinson, in *Proceedings 2nd Eso-Cern Symposium*, G.

Setti and L. van Hove, eds., p. 165 (1986).
45. G. Börner, *The Early Universe–Facts and Fiction* (Springer, Berlin, 1988), p. 44 ff.
46. G. Cocconi and E. E. Salpeter, *Nuovo Cimento* **10**, 646 (1958); *Phys. Rev. Lett.* **4**, 176 (1960).
47. V. W. Hughes, H. G. Robinson, and V. Beltran–Lopez, *Phys. Rev. Lett.* **4**, 342 (1960).
48. R. W. P. Drever, *Phil. Mag.* **6**, 683 (1961).
49. J. B. Barbour, *Br. J. Phil. Sci.* **33**, 251 (1982).
50. C. W. Allen, *Astrophysical Quantities* (Athlone, London, 1973), 3rd edn.
51. P. Claverie *et al.*, *Nature* **293**, 443 (1981).
52. R. Howard, *Ann. Rev. Astron. Astrophys.* **22**, 131 (1984).
53. H. Goldstein, *Classical Mechanics* (Addison–Wesley, Reading, 1980) 2nd edn.
54. R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw–Hill, New York, 1975) 2nd edn.
55. *General Relativity – An Einstein Centenary Survey*, S. W. Hawking and W. Israel, eds. (Cambridge University Press, London, 1979).
56. Ref. 22, article 528.
57. H. G. Grassmann, *Poggendorf's Ann. Phys. Chem.* **64**, 1 (1845).
58. Ref. 24, pp. 82-87.
59. R. A. R. Tricker, *Early Electrodynamics - The First Law of Circulation* (Pergamon, New York, 1965), pp. 55-58.
60. P. Graneau, *Nature* **295**, 311 (1982).
61. P. Graneau, *J. Appl. Phys.* **53** (10), 6648 (1982).
62. P. Graneau, *J. Phys. D* **20**, 391 (1987).
63. P. Graneau, *J. Appl. Phys.* **62** (7), 3006 (1987).
64. P. Graneau, *Phys. Lett.* **97 A** (6), 253 (1983).
65. P. Graneau, *J. Appl. Phys.* **55** (6), 2598 (1984).
66. P. Graneau, *Phys. Lett.* **107 A** (5), 235 (1985).
67. J. Nasilowski, *Phys. Lett.* **111 A** (6), 315 (1985).
68. H. Aspden, *Phys. Lett. A* **120** (2), 80 (1987).
69. P. Graneau, *Phys. Lett. A* **120** (2), 77 (1987).
70. P. Graneau and P. N. Graneau, *Appl. Phys. Lett.* **46** (5), 468 (1985).
71. R. Azevedo, P. Graneau, C. Millet, and N. Graneau, *Phys. Lett. A* **117** (2), 101 (1986).
72. H. Aspden, *IEEE Trans. Plasma Sci.* **PS–14** (3), 282 (1986).

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73. P. T. Pappas, *Phys. Lett.* **111 A** (4), 193 (1985).
74. P. G. Moyssides and P. T. Pappas, *J. Appl. Phys.* **59** (1), 19 (1986).
75. P. Graneau, *Nuovo Cimento* **78 B** (2), 213 (1983).
76. Parry Moon and D. E. Spencer, *J. Franklin Inst.* **257**, 203 (1954); *ibid.*, p. 305; *ibid.*, p. 369.
77. P. Graneau, *Hadronic J.* **10**, 145 (1987).
78. J. Hough *et al.*, *Vistas Astron. (UK)* **30** (2), 109 (1987).
79. P. A. M. Dirac, *Directions in Physics*, H. Hora and J. R. Shepanski, eds. (John Wiley, New York, 1978), p. 78.

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