International Journal of Theoretical Physics, Vol. 31, No. 6, 1992

Int. J. Theor. Phys. 31, 1063-1073 (1992)

The Ultimate Speed Implied by Theories of Weber's Type

A. K. T. Assis[1] and R. A. Clemente[2]

Received February 22, 1990

As in the last few years there has been a renewed interest in the laws of Ampère for the force between current elements and of Weber for the force between charges, we analyze the limiting velocity which appears in Weber's law. Then we make the same analysis for Phipps' potential and for generalizations of it. Comparing the results with the relativistic calculation, we obtain that these theories can yield c for the ultimate speed of the charges or for the ultimate relative speed between the charges but not for both simultaneously, as in the case in the special theory of relativity.

The question of an ultimate speed in nature is an old and puzzling one. Usually one thinks of light as the fastest body in the universe so that no material body could surpass its velocity. In the old days some thought the opposite. Consider this passage from Lucretius (circa 55 B.C.): "Obviously therefore they [free atoms in empty space] must far outstrip the sunlight in speed of movement and traverse an extent of space many times as great in the time it takes for the sun's rays to flash across the sky" (Lucretius, 1951, pp. 64–65).

In modern times ideas of this sort coupled with the special theory of relativity gave rise to the theory of tachyons (Recami, 1987; Mignami and Recami, 1988).

The goal of this paper is not to develop ideas related to tachyons, but to investigate the limiting velocity which appears naturally in W. Weber's theory and in some generalizations of it and then to compare these results

[1]Departamento de Raios Cósmicos e Cronologia, Instituto de Física, Universidade Estadual de Campinas, 13081 Campinas, SP, Brazil.

[2]Departamento de Eletrônica Quântica, Instituto de Física, Universidade Estadual de Campinas, 13081 Campinas, SP, Brazil.

with the relativistic ones. The theory of relativity is very well verified experimentally and so any study of this kind must be compared with the relativistic results. In this paper we will restrict ourselves to radial motion.

Weber's theory (Weber, 1846, 1848, 1893) states that the force on a charge q_i due to a charge q_j is given by

$$\mathbf{F}=\frac{q_i q_j}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1-\frac{\dot{r}^2}{2c^2}+\frac{r\ddot{r}}{c^2}\right) \tag{1}$$

where $\mathbf{r}\equiv\mathbf{r}_i-\mathbf{r}_j$, $r\equiv|\mathbf{r}|$, $\hat{r}\equiv\mathbf{r}/r$, $\dot{r}\equiv dr/dt$, $\ddot{r}\equiv d^2r/dt^2$, and c is the light velocity in vacuum. There has been a renewed interest in this law in recent years (Assis, 1989*a*,*b*; Wesley, 1987) because this is a powerful force law. Its main characteristics are: (a) it follows Newton's action and reaction law in the strongest form so that we have conservation of linear and angular momentum; (b) it can be derived from a velocity-dependent potential energy and follows the conservation of energy (Maxwell, 1954); (c) in a static situation ($\dot{r}=\ddot{r}=0$) we recover Coulomb's law; (d) Faraday's law of induction for closed circuits can be *derived* from Weber's law (Maxwell, 1954); (e) as it only depends on r, $\dot{r}$, and $\ddot{r}$, it is completely relational in its nature and so it has the same value for any observer even if the observer is noninertial (Assis, 1989*b*; Barbour and Bertotti, 1977, 1982); (f) Ampère's law for the force between two current elements (Ampère, 1825, 1958); can be *derived* from Weber's law (Maxwell, 1954). As a matter of fact, Weber devised equation (1) in order to get Ampère's law, which is

$$d\mathbf{F}=-\frac{\mu_0}{4\pi}I_1I_2\frac{\hat{r}}{r^2}[2d\mathbf{l}_1\cdot d\mathbf{l}_2-3(\hat{r}\cdot d\mathbf{l}_1)(\hat{r}\cdot d\mathbf{l}_2)] \tag{2}$$

To get equation (2) from equation (1) we only need to use the definition $I_i\,d\mathbf{l}_i\equiv q_{i+}(\mathbf{v}_{i+}-\mathbf{v}_{i-})$ and to suppose that $q_{i-}=-q_{i+}$ (i.e., that we are dealing with neutral current elements). Maxwell showed in his *Treatise* how the famous Ampère circuital law can be derived from equation (2). In the *Treatise* Maxwell said that equation (2) "must always remain the cardinal formula of electrodynamics" (Maxwell, 1954, Article 528).

One of the reasons why Weber's law is being revived is exactly the fact that equation (2) can be derived from it. Despite Maxwell's advice, the most widely used law of force between current elements is Grassmann's (1845) law sometimes known as Biot-Savart's law, which is

$$\begin{aligned}d\mathbf{F}&=\frac{\mu_0}{4\pi}\frac{I_1I_2}{r^2}d\mathbf{l}_1\times(d\mathbf{l}_2\times\hat{r})\\&=-\frac{\mu_0}{4\pi}\frac{I_1I_2}{r^2}[(d\mathbf{l}_1\cdot d\mathbf{l}_2)\hat{r}-(d\mathbf{l}_1\cdot\hat{r})\,d\mathbf{l}_2]\end{aligned} \tag{3}$$

In recent years many experiments have been made in order to differentiate between equations (2) and (3) when dealing with a single current so that we can decide (if it is possible to distinguish them experimentally) which is the best one in these situations (Graneau, 1982*a*,*b*, 1983, 1984, 1985, 1986, 1987*a*,*b*, 1989*a*,*b*; Graneau and Graneau, 1985, 1986; Pappas, 1983, 1985; Moyssides and Pappas, 1986; Nasilowski, 1984, 1985). These experiments are carried out with a single circuit because the two laws give the same value of the force on $I_1\, d\mathbf{l}_1$ when they are integrated over the entire circuit 2 if $I_1\, d\mathbf{l}_1$ is not a part of circuit 2 (Tricker, 1965, pp. 55–58). It should be emphasized here that although most of these experiments indicate that equation (2) is more correct than equation (3) when applied to a single circuit, this is still an open question and a growing and exciting scientific controversy surrounds the subject (Ternan, 1985*a*–*c*, 1986; Aspden, 1985*a*,*b*, 1986, 1987; Christodoulides, 1987; Whitney, 1988; Peoglos, 1988; Wesley, 1988; Cornille, 1989).

Before considering the limiting velocity which appears in Weber's law it is important to analyze the subject from the point of view of classical electromagnetism and see how it is modified by special relativity. We then consider two bodies of charges q_1 and q_2 and of masses m_1 and m_2 attracting each other. Neglecting radiation effects and the gravitational force as compared with the electrical one we obtain for this conservative system:

$$U + T = E = \text{const} \qquad \text{or} \qquad \frac{\alpha}{r} + \frac{M v_{cM}^2}{2} + \frac{\mu v^2}{2} = E \tag{4}$$

where

$$\alpha = \frac{q_1 q_2}{4\pi\varepsilon_0}$$

$M = m_1 + m_2$ is the total mass, $\mu = m_1 m_2 / M$ is the reduced mass, U is the electrical potential energy, T is the kinetic energy, E is the total energy for this system of two charges, $r = |\mathbf{r}_1 - \mathbf{r}_2|$, $\mathbf{v}_{cM} = (m_1\mathbf{v}_1 + m_2\mathbf{v}_2)/M$ is the velocity of the center of mass, and $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ is the relative velocity between the two charges. The total linear momentum of this system is also conserved in this case. In a reference frame in which the center of mass is at rest ($\mathbf{v}_{cM} = 0$) we get

$$\frac{\alpha}{r} + \frac{\mu v^2}{2} = E \tag{5}$$

This equation expresses the fact that the motion of charge 1 as viewed from charge 2 is the same as if charge 2 were fixed and charge 1 had a mass μ. Equation (5) shows that when r tends to zero, v tends to infinity. So classical electromagnetism without the relativistic corrections leads to the wrong result of a limitless boundary for the charge velocities.

In order to analyze this problem correctly, we need to include relativistic corrections. The main point here is that the mass of a moving particle is no longer a constant but should vary according to $m = m_0/(1 - v^2/c^2)^{1/2}$, where m_0 is the rest mass of this particle. This is also reflected in the kinetic energy of this particle, which in relativity theory turns out to be

$$T = \frac{m_0 c^2}{(1 - v^2/c^2)^{1/2}} - m_0 c^2 \tag{6}$$

Writing an equation similar to (4) yields

$$\frac{\alpha}{r} + \left(\frac{m_{01} c^2}{(1 - v_1^2/c^2)^{1/2}} - m_{01} c^2\right) + \left(\frac{m_{02} c^2}{(1 - v_2^2/c^2)^{1/2}} - m_{02} c^2\right) = E \tag{7}$$

The solution of this equation (and the one for the conservation of the total linear momentum) gives $v_1 = c$ and $v_2 = c$ when $r \to 0$. The modified addition law of velocities valid in special relativity theory gives in this case $v_1 - v_2 = c$. From this simple example it is seen that the ultimate velocity implied by relativity theory is in fact c, the light velocity. This is confirmed by all experiments with linear and circular accelerators, which are unable to accelerate any charged particle to a velocity greater than c. Expression (6) for the relativistic kinetic energy of a particle is a known experimental result. As an example, we cite the beautiful experiment of Bertozzi (1964), which was the basis for the teaching film, "The Ultimate Speed" (Angotti *et al.*, 1978).

Due to the renewed interest in Weber's law, we decided to study this problem from the point of view of Weber's theory. Weber showed that his force law, equation (1), could be derived from a generalized potential energy given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{\dot{r}^2}{2rc^2}\right) \tag{8}$$

He also showed the conservation of energy with this potential. Making a similar analysis as the one given in equations (4) and (5) yields

$$\frac{\alpha}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right) + \frac{\mu \dot{r}^2}{2} = E \tag{9}$$

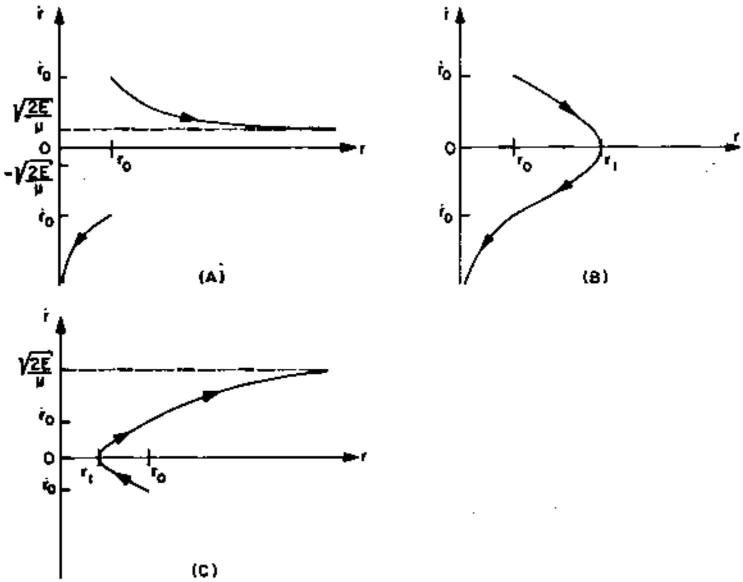

Fig. 1. Coulomb's law. (A) $\alpha<0$, and $E\geq 0$; (B) $\alpha<0$ and $E<0$; (C) $\alpha>0$, $E>0$.

From this equation it follows immediately that $\dot r \to -(2c)^{1/2}$ when $r\to 0$ (the minus sign appears due to the fact that we are considering an attraction between charges). In the center-of-mass coordinate frame we get

$$\begin{aligned} \mathbf{v}_1 &= -m_2\dot r\hat{\mathbf{r}}/M \\ \mathbf{v}_2 &= +m_1\dot r\hat{\mathbf{r}}/M \end{aligned} \tag{10}$$

Supposing $m_1=m_2=m$ (as when an electron is attracting a positron and vice versa), we obtain that although $|\dot r|>c$, $|\mathbf{v}_1|$ and $|\mathbf{v}_2|$ are smaller than c. The most important result of this section is to show how can we get an ultimate velocity near c in a classical theory like Weber's.

We present a graphical study of equations (5) and (9) in Figures 1 and 2. Using the definitions

$$\begin{gathered} \dot r_0 \equiv \dot r(t=0), \qquad r_0\equiv r(t=0) \\ \mu\equiv\frac{m_1m_2}{m_1+m_2}, \qquad \alpha\equiv\frac{q_1q_2}{4\pi\varepsilon_0} \\ A\equiv\pm\left(\frac{2E}{\mu}\right)^{1/2}, \qquad r_1\equiv\frac{\alpha}{E}, \qquad r_2\equiv\frac{\alpha}{\mu c^2} \end{gathered} \tag{11}$$

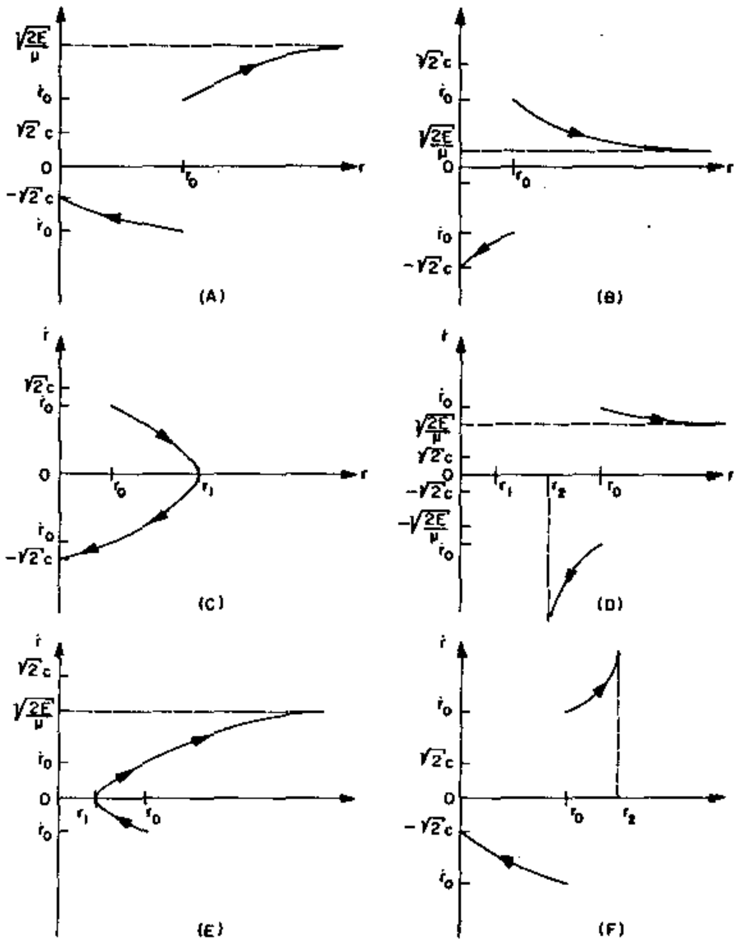

Fig. 2. Weber's law. (A) $\alpha<0$ and $E>\mu c^2$; (B) $\alpha<0$ and $0\leq E<\mu c^2$; (C) $\alpha<0$, $E<0$, $r_0\leq r_1$ and $\alpha>0$, $E>\mu c^2$, $r_2>r_1\geq r_0$; (D) $\alpha>0$, $E>\mu c^2$ and $r_0>r_2>r_1$; (E) $\alpha>0$, $0<E<\mu c^2$, and $r_2<r_1<r_0$; (F) $\alpha>0$, $0<E<\mu c^2$, $r_0<r_2<r_1$ and $\alpha>0$, $E\leq 0$, $r_0<r_2$.

we have for the Coulomb law

$$\dot{r}=A\left(\frac{r-r_1}{r}\right)^{1/2}$$

$$\ddot{r}=\frac{\alpha}{\mu r^2} \qquad \text{if} \quad r\neq r_1 \tag{12}$$

For Weber's law we have

$$\dot{r} = A\left(\frac{r - r_1}{r - r_2}\right)^{1/2}$$

$$\ddot{r} = \frac{\alpha}{\mu}\frac{1 - E/\mu c^2}{(r - r_2)^2} \tag{13}$$

In Figures 1 and 2 the arrows indicate the direction of motion. In the Coulomb case there is no real solution when $\alpha > 0$ and $E \leq 0$. From equation (13) we can see that there is no real solution also when $\alpha < 0$, $E < 0$, and $r_0 > r_1$; $\alpha > 0$, $E > \mu c^2$, and $r_2 > r_0 > r_1$; $\alpha > 0$, $0 < E < \mu c^2$, and $r_2 < r_0 < r_1$; and $\alpha > 0$, $E \leq 0$, and $r_0 > r_2$.

From these two figures we can see that the reflection point, $r = r_1$, is the same in both theories. The main differences are as follows: In Weber's theory one charge will not "feel" the other if they are moving relative to one another with a relative velocity $\pm\sqrt{2}c$, as this velocity will be constant in time. We do not have any equivalent in Coulomb's theory for the case when $|\dot{r}| > \sqrt{2}c$ in Weber's model. In this case, as can be seen from Figures 2a, 2d, and 2f, the relative velocity between the charges will always remain greater than $\sqrt{2}c$. Although we could compare these situations with the tachyonic case of relativity theory (which happens when $v > c$ instead of $\dot{r} > \sqrt{2}c$), we will not develop these analogies here, as the main goal of this paper is to analyze the ultimate speed of particles which move at a velocity less than or equal to $\sqrt{2}c$. We only point out here that the relative velocity for these superluminal particles ($\dot{r} > \sqrt{2}c$) would diverge if they came close enough to one another ($r = r_2 = 2 \times$ classical radius of the electron if $m_1 = m_2 = m_e$ or $m_1 = m_e$ and $m_2 \gg m_e$, and $q_1 = q_2 = e$), according to Weber's law.

The equivalents to Figures 1a–1c are Figures 2b, 2c, and 2e, respectively. Cases 1c and 2e are essentially the same. The main differences between Coulomb's law and Weber's law appear for slow charges (i.e., $\dot{r} < \sqrt{2}c$) when they approach each other. In this case $|\dot{r}| \to \infty$ according to Coulomb (Figures 1a and 1b), while $|\dot{r}| \to \sqrt{2}c$ according to Weber (Figures 2b and 2c).

We now analyze the same problem with the Phipps potential. This is a potential energy proposed by T. E. Phipps, Jr. (1990) in order to overcome Helmholtz' objection to Weber's law (von Helmholtz, 1870, 1872, 1873, 1874, 1875; Miller, 1981, pp. 92–93), as it is free of the "negative mass behavior" (Whittaker, 1973, pp. 198–205, 233–235) for all velocities smaller than c. As a matter of fact, we should mention here that Helmholtz was the first to study the ultimate speed implied by Weber's law. The Phipps potential is free of the instabilities which could occur in Weber's theory. It is given by

$$U=\frac{q_1q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1-\frac{\dot{r}^2}{c^2}\right)^{1/2} \tag{14}$$

It reduces to Weber's potential in second order of $\dot{r}/c$. It seems to indicate a limit of validity for Weber's theory: Weber's law can only be applied for situations which involve velocities only up to second order, inclusive, in $\dot{r}/c$. Assis (1988*a*) had already noted this limitation of Weber's law in a different context. Another limitation of Weber's law has long been known; namely, the fact that it is an action-at-a-distance theory. To overcome this obstacle and to get radiation effects in a theory derived from Weber's, workers have tried to introduce retarded action in Weber's law (Wesley, 1987; Moon and Spencer, 1954). In this paper we will not discuss these developments.

Returning to equation (14) and making a similar analysis which led to equation (9) leads us to

$$\frac{\alpha}{r}\left(1-\frac{\dot{r}^2}{c^2}\right)^{1/2}+\frac{\mu\dot{r}^2}{2}=E$$

or (15)

$$\dot{r}^4+\frac{4\alpha^2}{\mu^2r^2}\left(\frac{1}{c^2}-\frac{\mu Er^2}{\alpha^2}\right)\dot{r}^2+\frac{4\alpha^2}{\mu^2r^2}\left(\frac{E^2r^2}{\alpha^2}-1\right)=0$$

From equation (15) we obtain that $\dot{r}\to -c$ when $r\to 0$. This is exactly what should be expected according to relativity theory. If we analyze an interaction between two charges of the same mass, we get $|\mathbf{v}_1|=|\mathbf{v}_2|=c/2$.

The main properties of equation (15) can be summarized as follows:

$$\lim_{r\to 0}\dot{r}=-c$$

$$\lim_{r\to\infty}\dot{r}=\left(\frac{2E}{\mu}\right)^{1/2} \tag{16}$$

$$\dot{r}=0 \qquad \text{for} \quad r=r_1$$

$$\dot{r}=\pm c \qquad \text{for any } r \quad \text{if} \quad E=\frac{\mu c^2}{2}$$

there is no real r for which $\dot{r}$ diverges. So we can see that with the Phipps potential the ultimate relative speed is c for all charges. From equation (14) we can see immediately that there is no meaning for $|\dot{r}|>c$.

We could try also a more generalized potential, namely

$$U=\frac{\alpha}{r}\left(1+\alpha_1\frac{\dot{r}^2}{c^2}+\alpha_2\frac{\dot{r}^4}{c^4}+\cdots+\alpha_n\frac{\dot{r}^{2n}}{c^{2n}}+\cdots\right) \tag{17}$$

In order to discuss more closely the Phipps proposal, we study the potential energy given by

$$U=\frac{\alpha}{r}\left(1-\frac{1}{2\beta}\frac{\dot{r}^2}{c^2}\right)^{\beta} \tag{18}$$

where for any $\beta \neq 0$ we have the coefficient of $\dot{r}^2/c^2$ as $-1/2$, as in Weber's theory ($\beta = 1$ for Weber and $\beta = 1/2$ for Phipps). Making an analysis similar to the preceding ones, we can show that the ultimate relative velocity when $r \to 0$ is given by, supposing $\beta > 0$, $\dot{r} = -(2\beta)^{1/2}c$. If $m_1 = m_2$, then $|\mathbf{v}_1| = |\mathbf{v}_2| = (2\beta)^{1/2}c/2$, so that if $\beta = 2$, we get $|\mathbf{v}_1| = |\mathbf{v}_2| = c$, as happens according to relativity theory.

Comparing these results with the relativistic one shows that we can get $\dot{r} = c$ or $|\mathbf{v}_1| = |\mathbf{v}_2| = c$ in theories which are extensions of Weber's, but that we cannot obtain both at the same time (that is, $\dot{r} = c$ and $|\mathbf{v}_1| = |\mathbf{v}_2| = c$), as happens in the special theory of relativity.

Moreover, one must emphasize the physical differences in these results. In the special theory of relativity the reason for an ultimate speed is that the mass of the body is a function of its velocity relative to an inertial observer, which is also reflected in the kinetic energy of the body [see equation (6)]. As $v \to c$, we get that $T \to \infty$, and not to $mc^2/2$, as would be expected in classical theory without the relativistic corrections. The electric force given by Coulomb's law is always acting between the charges, even when $r \to 0$. As $m \to \infty$, when $v \to c$ we get that the acceleration of the body tends to zero and this gives an ultimate speed.

On the other hand, in the theory of Weber and in extensions of it, the reason for an ultimate speed is not a change in the kinetic energy, but a change in the potential energy [see equations (8), (11), and (14)]. In the case of Weber, for instance, what happens is that when $\dot{r} \to -\sqrt{2}c$, the force tends to zero, as can be seen from (1) with $r \to 0$. In this case the reason for an ultimate speed is a decrease in the electrical force and not a change in the inertial mass.

In conclusion, one can say that although we get similar results in all these theories, there are mathematical and physical differences between them. So they are not equivalent to one another.

ACKNOWLEDGMENTS

One of the authors (A.K.T.A.) wishes to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de

Desenvolvimento Cientifico e Tecnológico (CNPq), Brazil, for financial support, and also Professor I. L. Caldas for useful discussions and suggestions.

REFERENCES

Ampère, A. M. (1925). *Memoirs de l'Academie des Sciences*, **6**, 175.
Ampère, A. M. (1958). *Theorie Mathematique des Phenomenes Electrodynamiques*, Albert Blanchard, Paris.
Angotti, J. A. P., Caldas, I. L., Delizoicov Neto, D., Rüdinger, E., and Pernambuco, M. M. C. A. (1978). *American Journal of Physics*, **46**, 1258.
Aspden, H. (1985*a*). *Physics Letters A*, **111**, 22.
Aspden, H. (1985*b*). *Nuovo Cimento Letters*, **44**, 689.
Aspden, H. (1986). *IEEE Transactions on Plasma Science*, **PS-14**, 282.
Aspden, H. (1987). *Physics Letters A*, **120**, 80.
Assis, A. K. T. (1989*a*). *Physics Letters A*, **136**, 277.
Assis, A. K. T. (1989*b*). *Foundations of Physics Letters*, **2**, 301.
Barbour, J. B., and Bertotti, B. (1977). *Nuovo Cimento B*, **38**, 1.
Barbour, J. B., and Bertotti, B. (1982). *Proceedings of the Royal Society A*, **382**, 295.
Bertozzi, W. (1964). *American Journal of Physics*, **32**, 551.
Christodoulides, C. (1987). *Journal of Physics A*, **20**, 2037.
Cornille, P. (1989). *Journal of Physics A*, **22**, 4075.
Graneau, P. (1982*a*). *Nature*, **295**, 311.
Graneau, P. (1982*b*). *Journal of Applied Physics*, **53**, 6648.
Graneau, P. (1983). *Physics Letters A*, **97**, 253.
Graneau, P. (1984). *Journal of Applied Physics*, **55**, 2598.
Graneau, P. (1985). *Ampère–Neumann Electrodynamics of Metals*, Hadronic Press, Nonantum.
Graneau, P. (1986). *Fortschritte der Physik*, **34**, 457.
Graneau, P. (1987*a*). *Journal of Physics D*, **20**, 391.
Graneau, P. (1987*b*). *Journal of Applied Physics*, **62**, 3006.
Graneau, P. (1989*a*). *Electronics & Wireless World*, **1989**(June), 556.
Graneau, P. (1989*b*). *Physics Letters A*, **137**, 87.
Graneau, P., and Graneau, P. N. (1985). *Applied Physics Letters*, **46**, 468.
Graneau, P., and Graneau, P. N. (1986). *Nuovo Cimento D*, **7**, 31.
Grassmann, H. G. (1845). *Poggendorf's Annalen Physik Chemie*, **64**, 1.
Jolly, D. C. (1985). *Physics Letters A*, **107**, 231.
Lucretius (1951). *On the Nature of the Universe*, transl. R. Latham, Penguin, Harmondsworth.
Maxwell, J. C. (1954). *A Treatise on Electricity and Magnetism*, Vol. 2, Dover, New York, Chapter XXIII.
Mignani, R., and Recami, E., eds. (1988). *Classical Tachyons: Source Books*, Vols. 1–3, Hadronic Press, Nonantum.
Miller, A. I. (1981). *Albert Einstein's Special Theory of Relativity*, Addison-Wesley, Reading, Massachusetts.
Moon, P., and Spencer, D. E. (1954). *Journal of the Franklin Institute*, **257**, 203, 305, 369.
Moyssides, P. G., and Pappas, P. T. (1986). *Journal of Applied Physics*, **59**, 19.
Nasilowski, J. (1984). *IEEE Transactions on Magnetics*, **20**, 2158.
Nasilowski, J. (1985). *Physics Letters A*, **111**, 315.
Pappas, P. T. (1983). *Nuovo Cimento*, **76B**, 189.
Pappas, P. T. (1985). *Physics Letters A*, **111**, 193.

Peoglos, V. (1988). *Journal of Physics D*, **21**, 1055.
Phipps, Jr., T. E. (1990). *Physics Essays*, **3**, 414.
Phipps, Jr., T. E. (1990). *Apeiron*, **8**, 8.
Recami, E. (1987). *Foundations of Physics*, **17**, 239.
Ternan, J. G. (1985*a*). *Applied Physics Communications*, **57**, 1473.
Ternan, J. G. (1985*b*). *Journal of Applied Physics*, **57**, 1743.
Ternan, J. G. (1985*c*). *Journal of Applied Physics*, **58**, 3639.
Ternan, J. G. (1986). *Physics Letters A*, **115**, 230.
Tricker, R. A. R. (1965). *Early Electrodynamics—The First Law of Circulation*, Pergamon, New York.
Von Helmholtz, H. (1870). *Zeitschrift für Mathematik*, **72**, 57.
Von Helmholtz, H. (1872). *Philosophical Magazine*, **44**, 530.
Von Helmholtz, H. (1873). *Zeitschrift für Mathematik*, **75**, 35.
Von Helmholtz, H. (1874). *Zeitschrift für Mathematik*, **78**, 273.
Von Helmholtz, H. (1875). *Monatsberichte Akademie der Wissenschaften zu Berlin*, **1875**, 400–418.
Weber, W. (1846). *Abhandlungen Leibnizens Gesellschaft* (Leipzig), p. 316.
Weber, W. (1848). *Poggendorf's Annalen*, **73**, 229 [English translation, in *Taylor's Scientific Memoirs*, V, p. 489 (1852)].
Weber, W. (1893). *Wilhelm Weber's Werke*, Vols. 1–6, Springer, Berlin.
Wesley, J. P. (1987). *Speculations in Science and Technology*, **10**, 47.
Wesley, J. P. (1988). *Journal of Physics D*, **22**, 849.
Whitney, C. K. (1988). *Physics Letters A*, **128**, 232.
Whittaker, E. T. (1973). *A History of the Theories of Aether and Electricity—The Classical Theories*, Humanities Press, New York.