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An Analysis of Phipps's Potential Energy

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ABSTRACT: *We discuss a modification of Weber's law proposed by T. E. Phipps. We calculate the energy and force on a charge moving inside and outside a capacitor according to Phipps's modification of Weber's electrodynamics. When Phipps postulated his potential energy he answered Helmholtz's criticism of Weber's law (the negative mass behaviour). But when we utilize Phipps's potential energy together with the classical kinetic energy this leads to an unphysical result.*

I. Introduction

Wilhelm Weber obtained his force between the charges q_i and q_j, Eq. (1), from Ampère's force between current elements and from Fechner's hypothesis (electric currents being due to equal quantities of positive and negative charges moving in a wire with equal velocities but in opposite directions), (1). Nowadays we know that Fechner's hypothesis is incorrect, as only the electrons move in a metallic conductor carrying an electric current. On the other hand we now know that Ampère's force between current elements can be derived from Weber's force, even without Fechner's hypothesis. We only need to assume the charge neutrality of the current elements. This means that Ampère's force will remain valid whatever the velocities of electrons and positive ions, even when the positive ions are fixed in the lattice of a solid metallic conductor carrying an electric current due to the motion of electrons (2).

In modern vectorial language Weber's force is given by

$$\mathbf{F}_{ji} = \frac{q_i q_j}{4\pi\varepsilon_0} \frac{\hat{r}_{ij}}{r_{ij}^2} \left(1 - \frac{\dot{r}_{ij}^2}{2c^2} + \frac{r_{ij}\ddot{r}_{ij}}{c^2} \right)$$

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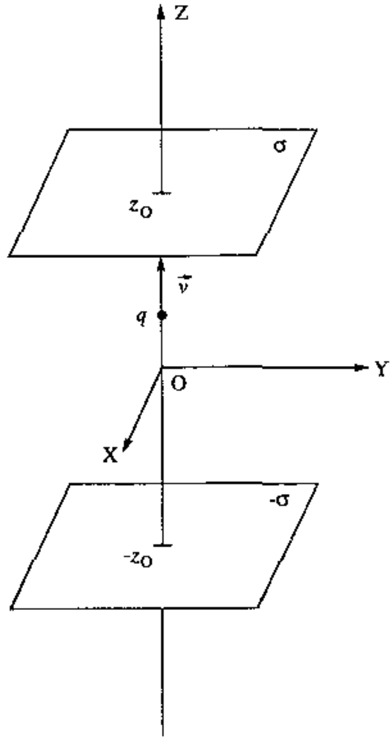

FIG. 1. Geometry of the problem.

$$= \frac{q_i q_j}{4\pi\varepsilon_0} \frac{\hat{r}_{ij}}{r_{ij}^2} \left[1 + \frac{\boldsymbol{v}_{ij} \cdot \boldsymbol{v}_{ij}}{c^2} - \frac{3}{2} \frac{(\hat{r}_{ij} \cdot \boldsymbol{v}_{ij})^2}{c^2} + \frac{\boldsymbol{r}_{ij} \cdot \boldsymbol{a}_{ij}}{c^2} \right] . \tag{1}$$

In this expression, $\boldsymbol{F}_{ji}$ is the force exerted by q_j on q_i, $\boldsymbol{r}_{ij} \equiv \boldsymbol{r}_i - \boldsymbol{r}_j$, $r_{ij} \equiv |\boldsymbol{r}_{ij}|$, $\hat{r}_{ij} \equiv \boldsymbol{r}_{ij}/r_{ij}$, $\boldsymbol{v}_{ij} \equiv \mathrm{d}\boldsymbol{r}_{ij}/\mathrm{d}t$, $\boldsymbol{a}_{ij} \equiv \mathrm{d}\boldsymbol{v}_{ij}/\mathrm{d}t$, $\dot{r}_{ij} \equiv \mathrm{d}r_{ij}/\mathrm{d}t$, $\ddot{r}_{ij} \equiv \mathrm{d}^2 r_{ij}/\mathrm{d}t^2$, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the vacuum permittivity, and c is the ratio of electromagnetic and electrostatic units of charge, which has the same value as the velocity of light in vacuum, namely, $c = 2.998 \times 10^8$ m/s.

This force can be derived from a velocity dependent potential energy given by Weber, namely,

$$U = \frac{q_i q_j}{4\pi\varepsilon_0} \frac{1}{r_{ij}} \left(1 - \frac{\dot{r}_{ij}^2}{2c^2} \right) . \tag{2}$$

In our previous work (3), we utilized these equations to study the motion of a charged particle moving orthogonally to the plates of an ideal capacitor with surface charge densities $\pm\sigma$ on the plates situated at $\pm z_0$ (Fig. 1).

When we integrated these equations on both plates for Weber's law (supposing an ideal capacitor with infinite plates) and added the classical kinetic energy, we obtained some interesting results. Weber's model coupled to Newtonian mechanics indicates (3) (1) a net force on the test charge outside the ideal capacitor whenever the test charge is accelerated by other bodies, (2) the test charge would move as if it had an effective inertial mass given by

$$m_{ei} = m + \frac{q\varphi(z)}{c^2}, \tag{3}$$

where m is the rest mass of the test particle, q its charge and $\varphi(z)$ is the classical electrostatic potential where the test charge is located (choosing $\varphi(z=0)=0$, so that $\varphi(z)=\sigma z/\varepsilon_0$ for $-z_0 < z < z_0$; and $\varphi(z \le -z_0) = -\sigma z_0/\varepsilon_0$; and $\varphi(z \ge z_0) = \sigma z_0/\varepsilon_0$); and (3) the velocity goes to infinity inside the capacitor when the effective inertial mass goes to zero. When the test particle is an electron this will happen whenever the voltage in the capacitor is larger than 1 MV.

On the other hand Weber's electrodynamics has many positive properties. It is simple, it obeys the laws of conservation of linear and angular momentum, and it can be derived from a velocity dependent potential energy. Therefore, it is consistent with the principle of conservation of energy. Moreover, Weber's force depends only on the separation of the two charges and its time derivatives. These aspects and some experimental results led some authors to propose some modifications on Weber's law (**2, 4–10**). Here we analyze Phipps's proposal, (**9**), in the situation studied in our previous work (**3**).

As in our previous paper, we will restrict our analysis to the speed of a charge moving inside a capacitor. We consider here the modification, proposed by Phipps, to Weber's potential energy together with the classical kinetic energy. The correct Lagrangian describing Phipps's interaction has been obtained recently by Bueno, (**11**).

The importance of this problem is the following: Weber's electrodynamics is a powerful model describing the interaction between point charges. When there is no motion between the charges we recover Coulomb's force and Gauss's law (the first of Maxwell's equations). From Weber's law we derive Ampère's force between current elements, from which Maxwell derived $\nabla \cdot \boldsymbol{B} = 0$ and "Ampère's circuital law" (two other of Maxwell's equations). Weber himself derived Faraday's law of induction from his force. For a proof and discussion of all these facts, see (**1**). However, we have shown that Weber's electrodynamics together with Newton's mechanics ($\boldsymbol{F} = m\boldsymbol{a}$ or $T = mv^2/2$) yields charges moving at velocities larger than the velocity of light, c (**3**). To overcome this limitation and preserve all positive aspects of Weber's electrodynamics we can modify Newtonian mechanics or Weber's electrodynamics for velocities close to c. In this paper we analyse this last possibility working with Phipps's potential energy, which reduces to Weber's one at second order in v/c. The relevance of Phipps's potential energy is that it overcame Helmholtz's criticism against Weber's electrodynamics, namely, the negative mass behaviour (**9**). This shows that it is worthwhile analysing Phipps's model in other contexts. This is the aim of the present work.

II. Phipps's Modification of Weber's Potential Energy and the Motion of a Charge Inside and Outside a Capacitor

According to Phipps, the factor 1/2 in Eq. (2) is highly suggestive. Consequently, he proposed the following postulate (9). The electromagnetic potential energy of two point charges q_i and q_j, with relative velocity $\dot{r}_{ij} = dr_{ij}/dt$, is given by

$$U_P = \frac{q_i q_j}{4\pi\varepsilon_0}\frac{1}{r_{ij}}\left(1 - \frac{\dot{r}_{ij}^2}{c^2}\right)^{1/2} . \tag{4}$$

Expanding this potential energy up to second order in $\dot{r}_{ij}/c$ yields Weber's potential energy, given by (2). Therefore Phipps's proposition is a modification of Weber's expression, which will only be relevant for high relative velocity $\dot{r}_{ij}$.

The interaction energy of a charge q moving along the z axis ($\boldsymbol{r} = z\hat{z}$, $\boldsymbol{v} = v\hat{z}$ and $\boldsymbol{a} = a\hat{z}$) in the situation described by Fig. 1 is obtained by integrating Eq. (4) for both plates. Supposing an ideal capacitor (with fixed charges in both plates whatever the motion of q) with infinite plates yields

$$U(\pm z - z_0 > 0) = \pm\frac{q\sigma z_0}{\varepsilon_0}\left[\sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c}\right] , \tag{5}$$

$$U(-z_0 \le z \le z_0) = \frac{q\sigma z}{\varepsilon_0}\left[\sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c}\right] . \tag{6}$$

The force on the test charge q due to the capacitor can be found by integrating Phipps's force, namely,

$$\begin{aligned}\boldsymbol{F}_P &= \frac{q_i q_j}{4\pi\varepsilon_0}\frac{\hat{r}_{ij}}{r_{ij}^2}\left[\left(1 - \frac{\dot{r}_{ij}^2}{c^2}\right)^{1/2} + \frac{r_{ij}\ddot{r}_{ij}}{c^2}\left(1 - \frac{\dot{r}_{ij}^2}{c^2}\right)^{-1/2}\right] \\ &= \frac{q_i q_j}{4\pi\varepsilon_0}\frac{\hat{r}_{ij}}{r_{ij}^2}\left[\sqrt{1 - \frac{(\hat{r}_{ij}\cdot\boldsymbol{v}_{ij})^2}{c^2}} + \frac{\boldsymbol{v}_{ij}\cdot\boldsymbol{v}_{ij} - (\hat{r}_{ij}\cdot\boldsymbol{v}_{ij})^2 + \boldsymbol{r}_{ij}\cdot\boldsymbol{a}_{ij}}{c^2\sqrt{1 - \frac{(\hat{r}_{ij}\cdot\boldsymbol{v}_{ij})^2}{c^2}}}\right] .\end{aligned} \tag{7}$$

After integration the net force is given by

$$\boldsymbol{F}(\pm z - z_0 > 0) = \pm\frac{q\sigma z_0}{\varepsilon_0}\frac{a}{cv}\left(\arcsin\frac{v}{c}\right)\hat{z} , \tag{8}$$

$$\boldsymbol{F}(-z_0 \le z \le z_0) = -\frac{q\sigma}{\varepsilon_0}\left[\sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c} + \frac{az}{cv}\arcsin\frac{v}{c}\right]\hat{z} . \tag{9}$$

Phipps's model, like Weber's model, indicates a net force on the test charge outside the ideal capacitor when the test charge is accelerated by other bodies. However, there are some differences. In Weber's model (Eq. (9) of (3)), we have a force whenever the test charge is accelerated; this force does not depend on the velocity of q. In Phipps's model, on the other hand, the force which acts upon the charge depends not only on its acceleration but also on its velocity.

Adding the classical kinetic energy to Eqs (5) and (6) yields the total energy E of the charge q:

$$E(z \le -z_0) = -\frac{q\Delta\varphi}{2}\left[\sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c}\right] + \frac{mv^2}{2} + K \tag{10}$$

$$E(-z_0 \le z \le z_0) = \frac{q\Delta\varphi}{2}\left[\sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c}\right]\frac{z}{z_0} + \frac{mv^2}{2} + K \tag{11}$$

$$E(z_0 \le z) = \frac{q\Delta\varphi}{2}\left[\sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c}\right] + \frac{mv^2}{2} + K\,. \tag{12}$$

In these expressions K is an arbitrary constant which can be given any value without affecting the results, and $\Delta\varphi = 2\sigma z_0/\varepsilon_0$ is the voltage between the two plates of the capacitor.

In this work we analyze the motion of a negative charge which is accelerated inside the capacitor beginning from rest at $z = -z_0$. Equating (10) and (11) yields

$$\frac{q\Delta\varphi}{2mc^2} = -\frac{\frac{v^2}{c^2}}{2\left\{\left[\sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}\arcsin\frac{v}{c}\right]\frac{z}{z_0} + 1\right\}}\,. \tag{13}$$

In Fig. 2 we plot the normalized velocity v/c of an electron ($q = -e$) as a function of its position inside the capacitor. The electron begins from rest at $z/z_0 = -1$ and increases its velocity as it moves towards the positive plate at $z/z_0 = 1$. These curves are obtained numerically from Eq. (13) for several values of the voltage $\Delta\varphi$ between the plates of the capacitor.

For $\Delta\varphi < 0.199$ MV the electron will leave the capacitor with a velocity smaller than c. When we increase the voltage beyond this value the electron will attain, according to this model, the velocity c at an intermediary point inside the capacitor. For instance, for $\Delta\varphi = 1$ MV the electron beginning from rest at $z = -z_0$ attains the velocity c inside the capacitor at $z = -0.3z_0$. If $\Delta\varphi$ tends to infinity the electron will reach the velocity c at $z = -0.64z_0$.

We do not analyze here the ranges in which $v/c > 1$, since there is no experimental evidence indicating that an electron can be accelerated beyond the light velocity. In Fig. 2 we see that for $\Delta\varphi > 0.2$MV the electron tends to overcome the light velocity inside the capacitor and this is clearly not borne out by experiments. For instance, in an important experiment due to Bertozzi, electrons were first accelerated inside a van der Graaf and later in the linear accelerator LINAC (12). The electrons attained energies of 0.5, 1.0, 1.5, 4.5 and 15 MeV. In all these cases the final velocities obtained directly through time-of-flight measurements never surpassed c. Clearly the findings of this experiment are not compatible with the predictions of Fig. 2 based on Phipps's potential energy coupled to Newtonian mechanics.

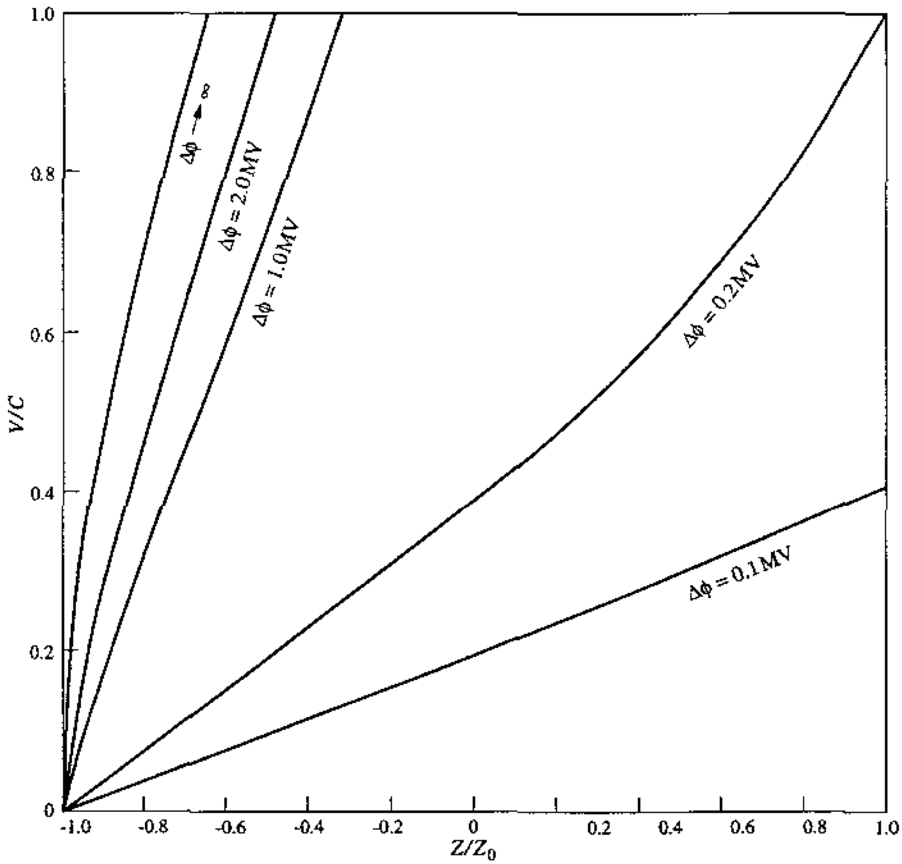

FIG. 2. Situations for Eq. (13) when the electrons moves orthogonally to the plates of the capacitor along the z axis, beginning from rest at $z = -z_0$.

III. Discussion and Conclusion

In this work we discussed Newton's mechanics ($\boldsymbol{F} = m\boldsymbol{a}$ or the kinetic energy given by $mv^2/2$) together with Phipps's modification of Weber's electrodynamics. This model leads to results which are not compatible with the experimental data. Nevertheless, we should indicate here all other assumptions which were implicitly and simultaneously utilized: we considered fixed charges in the capacitor while the test charge is moving through it, we did not include energy losses due to induced currents in the plates of capacitor, and losses due to radiation.

Electromagnetic radiation is not present in Weber's original electrodynamics nor in Phipps's model, so that it is fair to neglect its effects here (we could not do this if we were dealing directly with Maxwell's electrodynamics). We can ignore the currents induced in the plates of the capacitor, provided these plates are made of a nonconducting material. In this case (a capacitor made of charged dielectric plates), the charges over its surface are fixed in the material of the plates. At least in this idealized situation it is reasonable to neglect the induced currents.

This work showed that there is no ultimate velocity with Phipps's potential energy and Newtonian mechanics. That is, for $\Delta\varphi > 0.2\,\mathrm{MV}$ the electron will attain velocities higher than c, and this prediction is contradicted by the experiments. And this is a clear and unambigous limitation of the model.

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