Can a Steady Current Generate an Electric Field?

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Abstract

We present the results for the electric and magnetic fields due to linear and circular current distributions according to Weber's theory. We show how the electric field predicted by Weber's law is compatible with the anomalous diffusion in plasmas. Finally, we discuss some modern experiments related to this topic and compare the results of these experiments with a prediction based on Weber's law.

Key words: Weber's law, Ampère's law, radial electric field, tokamak transport, anomalous diffusion in plasmas, ambipolar diffusion, Phipps's potential, the motional electric field of Weber's law is a possible explanation of the anomalous diffusion in plasmas

1. INTRODUCTION

In the last few years there has been a renewed interest in the laws of Ampère and Grassmann (sometimes known as the Biot-Savart law) for the force between current elements[(1)–(3)] and in the law of Weber for the force between point charges.[(4)–(6)] As is well known, Weber's force law yields Ampère's empirical force law for the force between current elements.[(7)] With this law the famous circuital law of Ampère was derived. So the recent controversy[(8)–(12)] surrounding the different predictions of the laws of Ampère and Grassmann when applied to a single circuit is extremely relevant to the status of Weber's law in terms of its compatibility with the experimental results. These discussions arose from experiments with the electrodynamic impulse pendulum,[(2),(13),(14)] railgun accelerators,[(1),(3),(15)–(17)] exploding wire phenomena,[(18)–(20)] and electrodynamic explosions in liquids.[(21)–(23)] The relevance of these topics to gaseous discharges and plasma physics has been pointed out by Nasilowski.[(24),(25)]

Although these experiments seem to favor Ampère's law over Grassmann's law, this is still an unsolved question and more research is necessary in this direction. The aim of this paper is to present analytical solutions of the fields around static currents in important geometries. We also discuss the relevance of Weber's law to plasma physics and discuss the experimental results of some authors who searched for an electric field due to a steady current.

2. FORCE DUE TO A RECTILINEAR CURRENT

According to Weber's law, the force of a charge q_2 on a charge q_1 is given by[(4)–(7)]

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\left[1+\frac{1}{c^2}\left(r\ddot{r}-\frac{\dot{r}^2}{2}\right)\right]$$
$$= \frac{q_1 q_2}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\left\{1+\frac{1}{c^2}\left[\mathbf{v}_{12}\cdot\mathbf{v}_{12}-\frac{3}{2}(\hat{r}\cdot\mathbf{v}_{12})^2+\mathbf{r}\cdot\mathbf{a}_{12}\right]\right\} \quad (1)$$

where

$$\mathbf{r}\equiv\mathbf{r}_1-\mathbf{r}_2, \qquad r\equiv|\mathbf{r}|, \qquad \hat{r}\equiv\mathbf{r}/r,$$

$$\dot{r}\equiv dr/dt, \qquad \ddot{r}\equiv d^2r/dt^2,$$

$$\mathbf{v}_{12}\equiv\mathbf{v}_1-\mathbf{v}_2, \qquad \mathbf{a}_{12}\equiv\mathbf{a}_1-\mathbf{a}_2,$$

$$c\equiv \text{light velocity.}$$

In the case of electrostatics ($\dot{r}=0, \ddot{r}=0$) we recover Coulomb's law from Eq. (1).

Faraday's law of induction for closed circuits can also be derived from Eq.

(1), and Weber's law is also consistent with the principle of conservation of energy, since it can be derived from a velocity-dependent potential energy.[7] Weber's potential can be written in the form[4]

$$U = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2}\right). \tag{2}$$

A possible way to overcome Helmholtz criticism[26)–(29] of Weber's law has been given recently by Phipps. He considers Weber's law as only an approximation valid up to second order in $\dot{r}/c$. As a better model for the potential energy of interacting charges he proposes[30]

$$U = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{c^2}\right)^{1/2}, \tag{3}$$

which reduces to Weber's potential if we go only to second order in $\dot{r}/c$. Since Phipps's potential is free of the "negative-mass behavior" for all velocities smaller than c, he answered Helmholtz's objection.

We had also concluded that Weber's law is only an approximation when we analyzed the Kaufmann-Bucherer experiments.[31] We showed that Weber's law can explain this experiment only up to second order in v/c, inclusive, but that it failed for higher orders. However, it must be emphasized that the precision of these experiments was not higher than the second order in v/c.[32),(33]

In the present paper we are considering phenomena only up to second order in v/c; therefore, we will concentrate on the original Weber's law, and we will not consider recent developments, such as Phipps's proposal.

We now utilize this expression to calculate the force on a charge q_1, with velocity $\mathbf{v}_1$ and acceleration $\mathbf{a}_1$, due to a steady current I_2, infinite along the z-azis. We suppose this current distribution to have a zero net charge ($q_{2-} = -q_{2+}$, $n_{2+} = n_{2-} \equiv n_2$, where n_2 is the density of charged particles). We then have

$$dq_{2+} = q_{2+} n_2 dV = q_{2+} n_2 \rho d\rho d\varphi dz,$$

$$dq_{2-} = -q_{2+} n_2 dV = -q_{2+} n_2 \rho d\rho d\varphi dz,$$

$$\mathbf{v}_{2+} = v_{2+}\hat{z},$$
$$\mathbf{v}_{2-} = v_{2-}\hat{z},$$

where we are assuming a symmetric distribution of current around the z-axis, and that the element of charge dq_2 is at a distance ρ from the z-axis. We are utilizing cylindrical polar coordinates and φ is the azimuthal angle.

The force on q_1 is then obtained by integration. First, we calculate the force on q_1 due to a current in a cylindrical shell of radius ρ and thickness $d\rho$,

$$d\mathbf{F} = \int_{\varphi=0}^{\varphi=2\pi} \int_{z=-\infty}^{z=+\infty} (d\mathbf{F}_{2-\text{ in }1} + d\mathbf{F}_{2+\text{ in }1})$$

$$= A_1 \frac{\alpha}{\rho_1^2} \{[-2v_{1z}(v_{2+} - v_{2-})\rho_1\hat{\rho}_1 + 2(x_1 v_{1x} + y_1 v_{1y})(v_{2+} - v_{2-})\hat{z}]$$
$$+ (v_{2+}^2 - v_{2-}^2)\rho_1\hat{\rho}_1\}, \tag{4}$$

where

$$A_1 \equiv \frac{q_1 q_{2+} n_2 \rho d\rho}{4\pi\epsilon_0 c^2},$$

$$\alpha = \begin{cases} 0, & \text{if } \rho_1 < \rho \\ \pi, & \text{if } \rho_1 = \rho \\ 2\pi, & \text{if } \rho_1 > \rho. \end{cases}$$

We can express this result as

$$d\mathbf{F} = q_1(\mathbf{v}_1 \times d\mathbf{B} + d\mathbf{E}_M), \tag{5}$$

where

$$d\mathbf{B} = \frac{\mu_0 q_{2+} n_2 \alpha\rho(v_{2+} - v_{2-})}{2\pi\rho_1} d\rho\hat{\varphi}_1,$$

$$d\mathbf{E}_M = \frac{\mu_0 q_{2+} n_2 \alpha\rho(v_{2+}^2 - v_{2-}^2)}{4\pi\rho_1} d\rho\hat{\rho}_1.$$

The magnetic field in Eq. (5) is exactly the same as that obtained in classical electromagnetism. So Weber's law is in agreement with the Lorentz force regarding the magnetic field. The difference between the two results arises due to a motional electric field predicted by Weber's law $d\mathbf{E}_M$, because no such field should exist according to classical electromagnetic theory, due to the fact that we are considering a steady current ($dI_2/dt = 0$). We can estimate such electric field considering a linear metalic conductor, so that ($v_{2+} = 0$, $\lambda_{2+} = 2\pi\rho d\rho n_2 q_{2+}$, $I_2 = \lambda_{2+} v_{2-}$, $\alpha = 2\pi$):

$$\mathbf{E}_M = -\frac{\mu_0 |I_2 v_{2-}|}{4\pi\rho_1} \hat{\rho}_1, \tag{6}$$

where v_{2-} is the drift velocity of the electrons in the wire. Wesley[5),(6] obtained Eq. (6) and showed its negligible effect in laboratory conditions (this electric field gives rise to forces around 10^{-8} dyn). This shows why it is so difficult to test the existence of such a field in a typical laboratory.

Due to the great significance of Eq. (5), we decided to apply it in a typical tokamak regime like the joint European torus (JET) or the tokamak Fontenay-aux-Roses (TFR). In this regime we can approximate the distribution of charge in the current by[34),(35]

$$n_2(\rho) = n_{20}(1 - \rho^2/a^2), \tag{7}$$

where n_{20} is the value of the density at the axis of the plasma, and the parameter a is the minor radius of the plasma. We utilize an approximation of linear toroid so that we can apply Eq. (5) to a toroidal geometry. This is justifiable in the TFR machine because $a \ll R$, R being the major radius of the toroid. Performing the integration from $\rho = 0$ to $\rho = a$ yields

$$\mathbf{F} = q_1[\mathbf{v}_1 \times \mathbf{B}(\rho_1) + \mathbf{E}_M(\rho_1)], \tag{8}$$

where

$$\mathbf{B}(\rho_1) = -\frac{\mu_0 n_{20} e}{2}\left(1 - \frac{\rho_1^2}{2a^2}\right)\rho_1 v_{2-}\hat{\varphi}_1,$$

$$\mathbf{E}_M(\rho_1) = -\frac{|v_{2-}\mathbf{B}(\rho_1)|}{2}\hat{\rho}_1,$$

and where $-e$ is the electron charge. We utilized the fact that the ion drift velocity is much smaller than the electron drift velocity.

From Eq. (8) we can see that the force due to the poloidal magnetic field and the magnitude of this magnetic field calculated by Weber's law are exactly the same as those expected from Lorentz law. In addition to this, we obtained a radial electric field which should not exist according to classical electromagnetic theory. This electric field is independent of the velocity of q_1 and is always pointing in the direction of the current. Due to this field we should expect the electrons to drift radially towards the wall, while the ions should concentrate at the center of the plasma. This anomalous diffusion (anomalous according to classical electromagnetism) is observed to happen in tokamak discharges.(36) This is one of the most serious problems facing the controlled thermonuclear fusion program. The origins of this anomalous transport have never been completely understood. As we show here, Weber's force is a possible source for this radial flux of charges.

From Eq. (8) and observing that the electric field is proportional to the modulus of the poloidal magnetic field, we should expect the force and the flux of electrons to rise rapidly towards the discharge boundary, an effect observed in all the measurements of the flux of electrons.(37) This outward flux is not explained by classical or neoclassical theories.(37) This can be seen if we note that the nonthermal velocity of the electrons, their drift velocity, is a velocity along the external toroidal magnetic field. Since this toroidal magnetic field is parallel to the mean velocity of the electrons, it will not exert any force on them. So their interaction with the poloidal magnetic field generated by the plasma current would, by Lorentz force, move the electrons to the center of the plasma (classical transport, cylindrical geometry(38),(39)). If we consider toroidal geometry (neoclassical transport(40)–(44)) the same should happen because the force on the electrons due to ∇B_T^2 where $\mathbf{B}_T$ is the external toroidal magnetic field, is also directed for the center of the plasma.

For the sake of clarity we present here the definitions of classical and neoclassical transport theories as given by Balescu(39),(44): "The classical theory covers the transport phenomena in a plasma, considered as a collection of charged particles interacting through binary collision, in the presence of straight, homogeneous and stationary magnetic and electric fields." On the other hand, the neoclassical theory covers the transport phenomena in a plasma "in the presence of an inhomogeneous and curved magnetic field." Any other kind of transport that is not explained by these two theories is called "anomalous transport." That classical and neoclassical theories are insufficient to explain tokamak transport data has long been known.(45),(46)

In any case, the net outward flux of electrons should happen only until the moment when an opposite electric field is produced which counterbalances the former electric field. After this only an ambipolar diffusion can occur.(47) As we saw, the electric field of Eq. (8) is a possible explanation for these phenomena, but of course much more research is necessary in this direction before any conclusion can be drawn. The most accepted model now to explain the anomalous transport in plasmas is based on theories of turbulence(48)–(50) (for an earlier work on anomalous transport see Ref. 51). Because the role of tokamak microturbulence in anomalous transport is not yet completely understood,(50) we think it is a valid idea to explore other mechanisms as the driving force behind the anomalous diffusion of electrons.

It should be remembered here that even the runaway effect(52)–(54) cannot explain this anomalous transport in tokamaks, because this effect is related to the component of the electric field parallel to the external toroidal magnetic field.

3. FORCE DUE TO A CIRCULAR CLOSED LOOP

In general, we can reproduce the Lorentz force due to a magnetic field utilizing Weber's law, so in this section we will concentrate only on the motional or velocity-dependent electric field. This is the name we give to the electric field predicted by Weber's law and which should act on a charge at rest relative to a macroscopic magnet or at rest relative to a current-carrying wire. It is different from the classical electric field because it is not generated by a net electric charge.

This motional electric field is due to neutral currents in which the positive and negative charges of the current move with different velocities (in modulus), as can be seen from Eq. (5). Although this electric field is obtained through Weber's law, it should be emphasized here that to Weber himself this electric field shouldn't exist. This is because to Weber each current element should consist, following Fechner assumptions, of a positive and a negative charge which move toward each other with the same velocity relative to the wire.(28) At the time of Weber's writings and even of Maxwell's *Treatise* (1873), the internal nature of a current was not understood. Now, we know that the positive charges in a current-carrying wire don't move and that only the negative charges are responsible for the current. So, if Weber's law correctly describes the interactions between electric charges (at least up to second order in v/c), we should expect this motional electric field to exist and in principle this can be tested experimentally.

One limitation of the calculations presented in the previous section is the fact that the fields were calculated in an idealized geometry (a straight current of infinite length). From Eq. (1) we can see that Weber's force law is dependent on the accelerations of the charges and then any curvature in the current can affect the results, even in a steady-state situation.

To investigate this question we calculated the force on a charge q_1 due to a current I_2 in the form of a circular closed loop of radius ρ. To simplify the calculations, without any loss in generality, we place the loop in the xy plane, with the z-axis along the axis of symmetry of the loop. The charge q_1 is placed at rest in $\mathbf{r}_1 = x_1\hat{x} + z_1\hat{z}$. As in the previous section, we consider a steady current without a net charge and neglect the ions' movement $(v_{2+} = 0)$, as is the case for a metallic wire. We then obtain for the force on q_1

$$\mathbf{F} = q_1 \mathbf{E}_{\mathrm{M}}, \tag{9}$$

where

$$\mathbf{E}_{\mathrm{M}} = -\frac{\mu_0 |I_2 v_{2-}| x_1}{4\pi} \int_0^{2\pi} d\varphi \times \left\{ \frac{(x_1\hat{x} + z_1\hat{z})\cos\varphi - \rho\cos^2\varphi\,\hat{x}}{[(x_1^2 + z_1^2 + \rho^2) - 2x_1\rho\cos\varphi]^{3/2}} - \frac{3}{2}x_1\rho\,\frac{(x_1\hat{x} + z_1\hat{z})\sin^2\varphi - \rho\hat{x}\cos\varphi\sin^2\varphi}{[(x_1^2 + z_1^2 + \rho^2) - 2x_1\rho\cos\varphi]^{5/2}} \right\}.$$

From this expression we can immediately see that for a charge placed in the axis of the loop, for any z_1, there will be no motional electric field. This is an important effect due to the curvature in the wire. We can also observe that the electric field is always in the plane containing $\hat{\rho}_1$ and $\hat{z}$. This electric field is also independent of the direction of the current.

Considering q_1 in the plane of the loop $(z_1 = 0)$ and generalizing to

$\mathbf{r}_1 = x_1\hat{x} + y_1\hat{y} = \rho_1\hat{\rho}_1$ yields

$$\mathbf{F} \simeq q_1 \frac{\mu_0 |I_2 v_{2d}| \rho_1}{4\rho^2}\hat{\rho}_1, \qquad \text{if } \rho_1 \ll \rho, \tag{10}$$

$$\mathbf{F} \simeq -q_1 \frac{\mu_0 |I_2 v_{2d}| \rho}{8\rho_1^2}\hat{\rho}_1, \qquad \text{if } \rho_1 \gg \rho. \tag{11}$$

This is the force that should act on a static charge in the plane of a circular loop with a steady current according to Weber's law.

We must now discuss some experiments related to this topic. As we will see they are important but contradictory in their findings at the moment. As we saw from Eqs. (6), (10) or (11), if this motional electric field exists, it should be of the order V_D^2/c^2, where V_D is the drift velocity of the electrons in the wire. To our knowledge the first experiments devised to make direct measurements of this second-order electric field were those of Edwards[55],[56] and Edwards *et al.*[57] The technique used was that of measuring the potential resulting from such an electric field instead of measuring the electric field itself. To enhance the small drift velocity and so detect the effect, Edwards *et al.* used superconducting wire. They observed a potential that seems to arise due to a real motional electric field of the same order of magnitude as that of Eq. (11). Moreover, they found that the superconducting Nb-Ti coil carrying a direct current becomes *negatively* charged on its surface as would be expected according to Eqs. (6), (10) or (11). In agreement with these equations they found also that the potential (or electric field) is proportional to I_2^2 (or V_D^2) and is independent of the direction of the current. So their experiments gave a great support to Weber's law, or to any law of this kind.

Interpreting Edwards' results as being due to a weak variation of a particle's charge with its velocity, Bartlett and Ward[58] made two experiments to test this hypothesis. In the first one they rotated a current-carrying solenoid inside a Faraday ice pail, and in the second one they cooled a metal block inside an ice pail (the idea here is that a velocity-dependent potential would cause a neutral metal block to "charge" as it is heated, because the mean speed of the conduction electrons would be raised this way). In both experiments they found no motional electric field. Their results are inconsistent with those of Edwards and Edwards *et al.*

In comparing these two results it should be emphasized here that they are not equivalent. In particular, Edwards *et al.* utilized steady currents in closed conductors at rest, while Bartlett and Ward utilized an alternating current rather than a direct one in their first experiment, and in their second experiment they considered the motion of the conduction electrons in a block of metal. This motion is known to be random and oscillatory. So their motion is not a direct one in any direction, and they are being constantly accelerated and deviated by collisions. We note these aspects because Weber's law, Eq. (1), depends not only on the velocities of the charges, but also on their accelerations. This cannot be neglected when we analyze accelerated motions or currents in curved wires. So, a more careful analysis of the experiments of Bartlett and Ward should also consider a weak variation of particle's charge with its acceleration according to Eq. (1), as we intend to present in a future work. [We are here following the interpretation of Bartlett and Ward. They embodied the velocity terms of Eq. (1) in q_2 so that they could consider q_2 as having a weak variation depending on its velocity. In the same way we can embody the acceleration terms of Eq. (1) in q_2 and so we can interpret q_2 as having a weak variation also depending on its acceleration). One example of this restrict analysis is in a paper by Curé.[59] When suggesting a modification of the Millikan oil drop experiment to test the existence of a motional electric field as that obtained by Edwards, they obtained [see Eq. (12) of Ref. 59] a force on a charged oil drop at rest in the axis of a circular coil that carries a steady current. As we saw in Eq. (9) and the subsequent analysis, if we calculate the Weber force on a charge at rest in the axis of the loop [$x_1 = 0$ in Eq. (9)] it will be zero, this being due to the acceleration term of Eq. (1). This shows how careful we need to be in situations that involve acceleration of the charges.

Bonnet[60] tried to clear the incompatibility of the experimental results of Edwards *et al.* and of Bartlett and Ward supposing that steady moving charges in a superconductor do not radiate even if they move in a circular orbit. In this way he could neglect the acceleration terms of the Lienard-Wiechert potentials in the situation of Edwards' experiment. So they explained the findings of Edwards *et al.* based on this hypothesis of a nonradiating electron in a superconductor plus the usual Maxwell equations.

Perhaps the situation could be settled in this way if it were not for the experiment of Sansbury.[61] He found a force between a net stationary charge on a metal foil and a steady electric current in a wire. This experiment does not seem to be compatible with those of Bartlett and Ward who didn't find such a force. The explanation of Bonnet cannot be used this time because Sansbury utilized a simple U-shaped copper conductor at room temperature in this experiment, and so no phenomenon of superconductivity is implied here. This experiment also does not seem to be compatible with that of Edwards *et al.*, because Sansbury found that the copper conductor becomes *positively* charged on its surface. At the moment we cannot offer any explanations for these measurements of Sansbury.

Summarizing, we can say that all these experiments seem to be contradictory: Edwards *et al.* found the wire becoming negatively charged [in conformity with Eqs. (10) or (11)], Bartlett and Ward didn't find any change in the charge of the wire, and Sansbury found the wire becoming positively charged [against the prediction of Eqs. (10) or (11)]. This experimental situation seems to be confused at the moment, and we need more experiments before any conclusion can be drawn.

We must now turn our attention to an important paper by Salingaros.[62] In this paper he showed that a motional electric field can be seen as a relativistic effect. The main point of his analysis is the use of the Lorentz transformation laws of the charge-current 4-vector,[63]–[65] and so he obtains in the laboratory frame an electrostatic term proportional to v_e^2/c^2, where v_e is the electron average velocity relative to the rest frame of a fluid element. At first sight it appears to be an equivalent result to our Eq. (4) for he obtains in the laboratory frame a net charge density given by $env_e^2/2c^2$, but some remarks must be made. First, he assumes the plasma to be electrically neutral, as in the analysis of this paper, but he neglects the interparticle interactions. So the force he obtains with his net charge density only appears in the presence of an external electric field, as can be seen in his Eq. (16), for $\mathbf{E}_0$ to him is an externally produced electric field. These are major differences as compared to this work, because the motional electric field obtained here arose from interparticle interactions in the absence of an external electric field. Second, he ignored corrections of order $|\mathbf{u}|^2/c^2$, where $\mathbf{u}$ is the frame velocity, and used the Lorentz factor γ to be approximately equal to one. In our work the force terms of order $\dot{r}^2/c^2$ were included from the beginning and were of prime importance in deriving the motional electric field. So care must be taken when comparing these two works, because the physical origin of the motional force is different in each case. In any event, it is relevant to see that different physical theories give rise to similar effects. This can shed some light not only in these theoretical models but also in the origin of the fields detected in the experiments discussed above. Classical Maxwell theory seems to predict a null motional electric field (see Ref. 57 for a discussion on this topic). This shows the importance of a clear analysis of all these facts.

We only considered steady-state currents in this work, so we did not include the retardation of time in Weber's law, as was done by Moon and Spencer.[66] Some similar or different proposals to contain time delays in theories of action-at-a-distance have been discussed elsewhere.[5],[67]–[70] Something of this kind should be done whenever we have an experiment that involves fast time-varying fields, as we pointed out in an earlier work in which we extended Weber's law to gravitation.[71] Although in this model we derived the proportionality between inertial and gravitational masses, without needing to postulate it, and presented a possible way to implement Mach's principle, this was still an action-at-a-distance theory, as has been pointed out by Graneau.[72] So the results of the present paper should not be applied in situations that involve fast time-varying electric currents. We hope to present in the future a model to overcome this limitation.

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Résumé

Nous présentons les résultats pour les champs électrique et magnétique dus à des distributions linéaires et circulaires de courant selon la théorie de Weber. Nous montrons que le champ électrique prévu par la loi de Weber est compatible avec la diffusion anomale dans les plasmas. Enfin nous discutons certaines expériences modernes relatives à cet argument et nous comparons les résultats avec une prédiction basée sur la loi de Weber.

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