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A limitation of Weber's law

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We present Weber's law and its main properties. We discuss its relation with the experiments of mass variation with velocity. Then we calculate the energy and force on a charge moving inside and outside a capacitor according to Weber's electrodynamics. We discuss the consequences of this relation, and in particular we show that in this model a charge could attain velocities larger than the light velocity in a limited space due do a finite and feasible voltage difference. As this has never been observed we conclude that Weber's electrodynamics should not be applied to charges moving near the light velocity.

1. Introduction

Recently many works have appeared in the literature dealing with Weber's law as applied to gravitation and electromagnetism [1–7]. The reasons are manifold. In the first place Weber's force follows Newton's action and reaction law in the strongest form and so it is compatible with the conservation of linear and angular momentum. It can be derived from a velocity dependent generalized potential energy and is consistent with the principle of conservation of energy. When applied to gravitation it yields the precession of the perihelion of the planets [7,8] and models the effect of a delay in the gravitational interaction of material bodies [2]. When applied to electromagnetism it yields Coulomb's law, Faraday's law of induction and Ampère's expression for the force between current elements [9,10]. Ampère's circuital law was originally derived by Maxwell through this last expression, which he called the cardinal formula of electrodynamics (ref. [9], §528). In the last few years many experiments have been performed trying to distinguish Ampère's force law from Grassmann's one (which is based on Biot–Savart's expression for the magnetic field), but this is still an open question [1,11–23]. Here it should only be emphasized that Maxwell knew Grassmann's force law but preferred Ampère's expression because it was the only one which satisfied Newton's third law in the strongest form (ref. [9], §§526, 527). It should be remarked here that even if it is proved that Ampère's force law is the only one compatible with the experimental results this will not vindicate Weber's force. The point is that Weber's force deals with point charges while Ampère's force deals with neutral current elements (many body system). Although we can derive Ampère's force from Weber's one performing a statistical summation over all interacting charges of the neutral current elements, this can also be done from other approaches and different force laws. For instance, recently Rambaut and Vigier succeeded in deriving Ampère's force from the relativistic limit of the Lorentz force on the macroscopic level [1]. As they deduced Ampère's force as a non-relativistic approximation of the sum of all Lorentz interactions acting on individual current elements, it can be said that Ampère's force law is compatible not only with Weber's electrodynamics but also with the standard Maxwell–Lorentz–Einstein electromagnetic theory.

Despite all these positive aspects there are some characteristics of Weber's law which give rise to some discussion. One of them is the electric field which

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should exist near a neutral, stationary and steady current according to Weber's force. As we already have discussed this topic [10] in conjunction with the experimental findings of Edwards et al., Bartlett and Ward and Sansbury [24–26], in this work we will restrict our analysis to another topic, the ultimate speed of a charge moving inside a capacitor. It points out, unambiguously, a limitation of Weber's law. This limitation, as we will see, indicates that Weber's law should not be applied to high speed electrons (as when we have isolated charges being accelerated in high energy accelerators). This is in agreement with Rambaut and Vigier who apply Weber's law only in a many-body situation (charges inside a neutral conductor), but never for free charges [1]. But first, let us discuss the relation of Weber's law with mass variation.

2. Weber's law and mass variation

In modern vectorial language Weber's force law can be written as [9,27,28]

$$\boldsymbol{F}_{ji}=\frac{q_i q_j}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{r}}_{ij}}{r_{ij}^2}\left(1-\frac{\dot{r}_{ij}^2}{2c^2}+\frac{r_{ij}\ddot{r}_{ij}}{c^2}\right)$$

$$=\frac{q_i q_j}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{r}}_{ij}}{r_{ij}^2}\left(1+\frac{\boldsymbol{v}_{ij}\cdot\boldsymbol{v}_{ij}}{c^2}-\frac{3}{2}\frac{(\hat{\boldsymbol{r}}_{ij}\cdot\boldsymbol{v}_{ij})^2}{c^2}+\frac{\boldsymbol{r}_{ij}\cdot\boldsymbol{a}_{ij}}{c^2}\right). \tag{1}$$

In this expression $\boldsymbol{F}_{ji}$ is the force exerted by q_j on q_i, $\boldsymbol{r}_{ij}\equiv\boldsymbol{r}_i-\boldsymbol{r}_j$, $r_{ij}\equiv|\boldsymbol{r}_{ij}|$, $\hat{\boldsymbol{r}}_{ij}\equiv\boldsymbol{r}_{ij}/r_{ij}$, $\boldsymbol{v}_{ij}\equiv \mathrm{d}\boldsymbol{r}_{ij}/\mathrm{d}t$, $\boldsymbol{a}_{ij}\equiv d\boldsymbol{v}_{ij}/dt$, $\dot{r}_{ij}\equiv \mathrm{d}r_{ij}/\mathrm{d}t$, $\ddot{r}_{ij}\equiv \mathrm{d}\dot{r}_{ij}/\mathrm{d}t$ and c is the ratio of electromagnetic and electrostatic units of charge, which has the same value as the velocity of light in vacuum. This force can be derived from a velocity dependent potential energy given by

$$U=\frac{q_i q_j}{4\pi\varepsilon_0}\frac{1}{r_{ij}}\left(1-\frac{\dot{r}_{ij}^2}{2c^2}\right). \tag{2}$$

In a previous paper we utilized eq. (1) to study the motion of a charge in a region of crossed electric and magnetic fields [29]. In particular we analysed the experiments of Kaufmann and Bucherer which were devised to test the variation of mass with velocity. We compared the results of relativity theory ($m=m_0/(1-v^2/c^2)^{1/2}$ plus Lorentz's force law) with Weber's model (without mass variation) and showed that the radius of the charge trajectory (which was measured experimentally) after it had left the capacitor was given by, respectively [29],

$$r_\mathrm{R}=\frac{\sigma_\mathrm{A}}{\varepsilon_0 e B^2}\,\frac{m_0}{(1-v_\mathrm{R}^2/c^2)^{1/2}}$$

$$=\frac{m_0\sigma_\mathrm{A}}{\varepsilon_0 e B^2}\left(1+\frac{v_\mathrm{R}^2}{2c^2}+\frac{3}{8}\frac{v_\mathrm{R}^4}{c^4}+\dots\right), \tag{3}$$

$$r_\mathrm{W}=\frac{m_0\sigma_\mathrm{A}}{\varepsilon_0 e B^2}\left(1+\frac{v_\mathrm{W}^2}{2c^2}\right). \tag{4}$$

In these expressions m_0 and $-e$ are the electron's mass and charge, σ_A is the surface charge density of the capacitor and B is the magnetic field in which the electrons move after they had left the capacitor. In (3) and (4) R refers to relativity and W to Weber's model.

A comparison of these two expressions led us to conclude that Weber's law is only an approximation, valid only up to second order in v/c, and that only special relativity was compatible with these experiments to all orders of v/c. But some remarks must be made. The first one is that Bush had arrived at a similar conclusion, although he utilized a force law of his own which was similar, but not identical, to Weber's force [30]. This problem was also discussed by O'Rahilly using Ritz's theory (ref. [28], pp. 249, 250, 613–622).

Wesley has shown that our conclusion is not correct [31,32]. He correctly pointed out that v_R and v_W which appear in (3) and (4) are not the same, as each one of them is different function of E_0 (the uniform electric field inside the capacitor) and B. From eqs. (9) and (15) of ref. [29] we have $v_\mathrm{R}=E_0/B=\sigma_\mathrm{A}/\varepsilon_0 B$ and $eE_0(1+v_\mathrm{W}^2/2c^2)=eBv_\mathrm{W}$, so that $v_\mathrm{W}=(c^2B^2/E_0)[1-(1-2E_0^2/c^2B^2)^{1/2}]$. (In our previous work [29] we did not utilize the subscripts R and W.) As E_0 and B are the same in both models and were measured directly in the experiments of Kaufmann and Bucherer, we should express (3) and (4) in terms of E_0 and B to compare them. When this is done we get [31,32]

$$r_R = \frac{m_0 E_0}{eB^2} \frac{1}{(1-E_0^2/c^2B^2)^{1/2}}$$
$$= \frac{m_0 E_0}{eB^2}\left(1+\frac{1}{2}\frac{E_0^2}{c^2B^2}+\frac{3}{8}\frac{E_0^4}{c^4B^4}+...\right), \qquad (5)$$

$$r_W = \frac{m_0 E_0}{eB^2}\left(1+\frac{1}{2}\frac{E_0^2}{c^2B^2}+\frac{1}{2}\frac{E_0^4}{c^4B^4}+...\right). \qquad (6)$$

As we can see also Weber's law has fourth and higher orders in E_0/cB. It should be remarked that the precision of the experiments of Kaufmann and Bucherer was not beyond the second order in v/c (or E_0/cB) [33,34].

This analysis indicates that these experiments on the variation of the electron's mass are not the best suited to distinguish between Weber's law and standard approaches. But here we present a specific example that shows clearly the limitations of Weber's law. It also involves a capacitor as in these experiments of mass variation, but it has no magnetic field.

3. Motion of a charge inside and outside a capacitor according to Weber's law

We discuss here the motion of a charged particle moving orthogonally to the plates of an ideal capacitor with surface charge densities $\pm\sigma$ on the plates situated at $\pm z_0$ (fig. 1). Classically there is no electric field outside the capacitor, but inside there is a uniform electric field given by $\boldsymbol{E}_0 = -\sigma\hat{\boldsymbol{z}}/\varepsilon_0$, which means that there is a voltage difference between the two plates given by $V_0 = 2\sigma z_0/\varepsilon_0 = 2z_0E_0$.

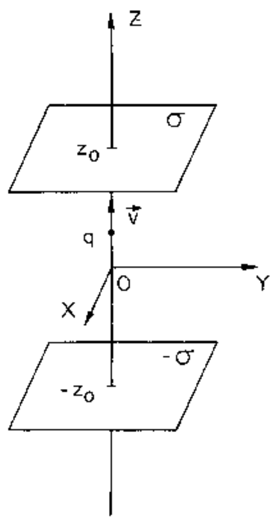

Fig. 1.

The interaction energy and force exerted on a charge q moving along the Z axis ($\boldsymbol{r}=z\hat{\boldsymbol{z}}$, $\boldsymbol{v}=v\hat{\boldsymbol{z}}$ and $\boldsymbol{a}=a\hat{\boldsymbol{z}}$) is obtained integrating eqs. (1) and (2) on both plates for Weber's law. The result of these integrations is given by (using an ideal capacitor with infinite plates)

$$U = \pm q\frac{\sigma}{\varepsilon_0} z_0\left(1+\frac{v^2}{2c^2}\right), \qquad \pm z - z_0 > 0, \qquad (7)$$

$$= q\frac{\sigma}{\varepsilon_0} z\left(1+\frac{v^2}{2c^2}\right), \qquad -z_0 \leqslant z \leqslant z_0; \qquad (8)$$

$$\boldsymbol{F} = \mp q\frac{\sigma}{\varepsilon_0}\frac{z_0 a}{c^2}\hat{\boldsymbol{z}}, \qquad \pm z - z_0 > 0, \qquad (9)$$

$$= -q\frac{\sigma}{\varepsilon_0}\left(1+\frac{v^2}{2c^2}+\frac{za}{c^2}\right)\hat{\boldsymbol{z}}, \quad -z_0 \leqslant z \leqslant z_0. \qquad (10)$$

The general expression for the force on a charge q moving in any direction inside a capacitor had been obtained previously [29]. Here we particularize for motions orthogonal to the plates, but include also situations outside the capacitor.

The first remark to be made is that besides changing the classical result for the force inside a capacitor, including terms which depend on the velocity and acceleration of the test charge, Weber's model indicates a net force on the test charge *outside* the ideal capacitor whenever it is accelerated. This is a completely novel result which is drastically different from Coulomb's law. Alternatively expressions (7)–(10) can be interpreted saying that according to Weber's model the inertial mass of a charge will be a function of the electrostatic potential where it is located. In the following we discuss this aspect in more detail.

Adding the kinetic energy to eqs. (7) and (8) yields (remembering that Weber's law is compatible with the principle of conservation of energy) for the total energy E of the charge q:

$$E(z<-z_0) = -\tfrac{1}{2}q_1V_0 + \tfrac{1}{2}(m-m_W^0)v^2 + k, \qquad (11)$$

$$E(-z_0 \leqslant z \leqslant z_0)$$
$$= -\tfrac{1}{2}q_1V_0z/z_0 + \tfrac{1}{2}(m+m_W)v^2 + k, \qquad (12)$$

$$E(z>z_0)=\tfrac{1}{2}q_1 V_0+\tfrac{1}{2}(m+m_W^0)v^2+k\,. \qquad (13)$$

In these expressions k is an arbitrary constant to which we can give any value without affecting the results, and $V_0\equiv 2\sigma z_0/\varepsilon_0=2z_0E_0$ is the voltage difference between the two plates. Moreover m_W^0 and m_W are what we call Weber's inertial masses. They are given by $m_W^0\equiv qV_0/2c^2$, and $m_W\equiv qV_0z/2c^2z_0$. If we choose the zero of the potential as being at $z=0$ (the middle point between the two plates), eqs. (11)–(13) would be the same as the classical result with m replaced by $m+qV(z)/2c^2$, where $V(z)$ is the classical electrical voltage where the test charge is located. This means that according to Weber's model the charge would move as if it had an effective inertial mass given by $m+qV(z)/2c^2$.

The simplest way to study this problem and to show the inadequacy of Weber's law to high velocities is to consider a charge q (for instance an electron, with $q=-e$) which comes from $z<-z_0$, moving towards the capacitor along the Z axis with $v=v_b$, being accelerated between the plates, and which leaves the capacitor with velocity v_a. From the conservation of energy we get, using (11) and (13),

$$v_a=\left[\frac{2}{m-|m_W^0|}\left(\frac{m+|m_W^0|}{2}v_b^2+eV_0\right)\right]^{1/2}. \qquad (14)$$

The terms inside the parentheses in (14) are all positive. This equation indicates that the electron will only pass the capacitor if $|m_W^0|<m$. If $|m_W^0|>m$ the electron will not arrive at the second plate. The limiting case is when $|m_W^0|=m$, which indicates that the electron will arrive at the second plate with an infinite velocity. The voltage necessary for this to happen is given by $V_0=2mc^2/e\approx 10^6$ V. As this voltage and higher ones have been produced in many high energy accelerators and the electron has never attained a velocity larger than c, this proves a serious limitation of Weber's law.

Another simple analysis is when the electron is generated inside the capacitor at $z=0$ with negligible initial velocity. According to (12) its velocity, as a function of the position inside the capacitor, is given by

$$v^2=\frac{2|m_W^0|c^2z}{mz_0-|m_W^0|z}, \quad -z_0\leqslant z\leqslant z_0\,. \qquad (15)$$

This shows that the electron can only move in the region $0\leqslant z<mz_0/|m_W^0|$. If $|m_W^0|=m$ the velocity will go to infinity at the second plate. If $|m_W^0|>m$ then there will be a divergence in the velocity at an internal point. If, for instance, $m_W^0=-2m/3$ the electron will leave the second plate with a velocity given by twice the value of light. Once more there are no known experiments in the literature which corroborate such findings.

As an experiment which gives results not compatible with these predictions based on Weber's law, we cite the beautiful one performed by Bertozzi many years ago in which he measured not only the speed of the relativistic electrons after they were accelerated through a linear accelerator but also their kinetic energy by calorimetry [35]. In this experiment electrons were generated inside a van der Graaff and later accelerated in the linear accelerator LINAC (at MIT), so that they acquired kinetic energies from 0.5 to 15 MeV. The limiting speed of the electrons was always found to be c.

4. Discussion and conclusion

In this work we have discussed that if we utilize Newton's mechanics (**F**=m**a**, or kinetic energy $=\frac{1}{2}mv^2$) plus Weber's electrodynamics we can explain the experiments of Kaufmann and Bucherer without mass velocity change [31,32]. On the other hand we have shown that Newton's mechanics plus Weber's electrodynamics leads to results never found experimentally (electrons attaining velocities larger than c). This shows clearly the inadequacy of this approach in situations which involve large values (≈ 1) of v/c. There seems to be no escape from this conclusion. Nevertheless we should indicate here all other assumptions which were implicitly utilized simultaneously: we considered the charges in the capacitor or accelerator to be fixed while the test charge is moving through it. In practice this is only an approximation as the test electrons should lose energy by inducing currents in the plates of the capacitor as they move through it. Moreover we did not include the losses due to radiation (the test electron is accelerated and so it should radiate). The point is that Weber's law does not include radiation in its formulation. This means that to include radiative losses we need to modify Weber's law which, once more, corroborates our conclusion that there are limitations in Weber's approach. A modification of Weber's law to include radiation was given by Moon and Spencer [36]. A similar approach was followed by Wesley [4,31,32], and a different way of obtaining time delays in action-at-a-distance theories was given by Graneau [37]. Here we will not discuss any of these approaches. The original Weber's law, as presented here, is an action-at-a-distance theory. Although it models a delay in the propagation of interactions [2], it cannot include all aspects of radiation. To follow Newton's action and reaction law in the strongest form and to be an action-at-a-distance theory, may be a positive aspect [38–40], but we will not discuss here these points since they are outside the scope of the present work.

A topic discussed in the paper was an alternative interpretation of these results of Weber's electrodynamics as indicating a variation of the inertial mass of a charged particle with electrostatic potential. This is somewhat similar to the gravitational redshift (a variation of mass with the gravitational potential) and reminds us of the Einstein mass–energy relation $E=mc^2$, because $m_W=qV/2c^2$ (the energy being here a potential energy). However it should be pointed out that this analogy has limitations because according to Weber's model the value of the effective mass will depend on the geometry of the problem and not only on the value of the potential. For instance if the charge were inside a spherical shell charged to the voltage V it would behave as if it had an inertial mass given by $m+qV/3c^2$ [9], and not $m+qV/2c^2$ as when it is inside of a capacitor (Weber's model). According to relativity theory there is no such dependence on the geometry. Anyway this effect predicted by Weber's theory is an essential part of the model. To our knowledge no experiment has been designed to test this effect. We propose it to be done with low velocity electrons due to the previous limitation we pointed out above. Any experiment involving positrons or electrons moving in regions of varying electrostatic potentials can be utilized to test this effect, provided the outcome of the experiment (i.e., what is measured involves the particle mass). The only experiment of this kind we know has been performed by Kennedy and Thorndike [41]. But as they utilized a photon as a test particle (moving through points varying by 50000 V) their null result cannot be utilized directly here because a photon has no net charge.

Recently Phipps proposed a modified Weber's potential to overcome Helmholtz's criticism of Weber's law [42]. Essentially he proposes a potential energy given by $U=(q_1q_2/4\pi\varepsilon_0 r_{12})(1-\dot{r}_{12}^2/c^2)^{1/2}$. As this is a modification of Weber's original potential we will not consider it here.

Anyway it should be emphasized that Helmholtz's criticism of Weber's law [9,42] has never been proved wrong. In particular he showed that according to Weber's model a charge could behave under some conditions as if it had a negative inertial mass or as having an equivalent inertial mass going to zero, even with non-relativistic velocities. To our knowledge this has never been observed experimentally, which casts some doubts on the feasibility of applying Weber's forces even for slow velocities. Although Phipps overcame Helmholtz's criticism [42], this was only possible modifying Weber's potential energy, which confirms once more the limitations of Weber's law. Our goal here was only to show a specific limitation of Weber's law: the prediction that charged particles can attain velocities as large as we wish, in a limited space, with finite voltage differences. Since this has not been confirmed by experiments we can conclude that Weber's law should not be applied to velocities near the light velocity.

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