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On the First Electromagnetic Measurement of the Velocity of Light by Wilhelm Weber and Rudolf Kohlrausch

Abstract

The electrostatic, electrodynamic and electromagnetic systems of units utilized during last century by Ampère, Gauss, Weber, Maxwell and all the others are analyzed. It is shown how the constant c was introduced in physics by Weber's force of 1846. It is shown that it has the unit of velocity and is the ratio of the electromagnetic and electrostatic units of charge. Weber and Kohlrausch's experiment of 1855 to determine c is quoted, emphasizing that they were the first to measure this quantity and obtained the same value as that of light velocity in vacuum. It is shown how Kirchhoff in 1857 and Weber (1857-64) independently of one another obtained the fact that an electromagnetic signal propagates at light velocity along a thin wire of negligible resistivity. They obtained the telegraphy equation utilizing Weber's action at a distance force. This was accomplished before the development of Maxwell's electromagnetic theory of light and before Heaviside's work.

1. Introduction

In this work the introduction of the constant c in electromagnetism by Wilhelm Weber in 1846 is analyzed. It is the ratio of electromagnetic and electrostatic units of charge, one of the most fundamental constants of nature. The meaning of this constant is discussed, the first measurement performed by Weber and Kohlrausch in 1855, and the derivation of the telegraphy equation by Kirchhoff and Weber in 1857. Initially the basic systems of units utilized during last century for describing electromagnetic quantities is presented, along with a short review of Weber's electrodynamics. An earlier discussion of these topics has been given.[1]

[1] Assis (2000a)

2. Forces of Nature

The first definition of Newton's book *Mathematical Principles of Natural Philosophy* of 1687, usually known by the first Latin name, *Principia*, is that of quantity of matter. He defined it as the product of the density and volume of the body. He says:

> It is this quantity that I mean hereafter everywhere under the name of body or mass.[2]

This magnitude is called nowadays the *inertial mass* of the body. His second definition is that of *quantity of motion*, the mass of a body times its velocity relative to absolute space. His third definition is that of inertia or force of inactivity:

> The vis insita, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.

His second law of motion states:

> The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

Representing this force in terms of vectors by $\vec{F}$, the inertial mass by m_i and the velocity of the body relative to absolute space or to an inertial frame of reference by $\vec{v}$, the second law can be written as

$$\vec{F} = K_1 \frac{d(m_i \vec{v})}{dt}, \tag{1}$$

where K_1 is a constant of proportionality.

According to the law of universal gravitation the force exerted by a gravitational mass $m_g{}'$ on another gravitational mass m_g separated by a distance r is given by

$$\vec{F} = -K_2 \frac{m_g m_g{}'}{r^2} \hat{r} \ . \tag{2}$$

Here K_2 is a constant of proportionality and $\hat{r}$ is the unit vector pointing from $m_g{}'$ to m_g. This force is along the straight line connecting the masses and is always attractive.

[2] NEWTON (1934).

The gravitational force on a particle of gravitational mass m_g due to other masses can be written as

$$\vec{F} = m_g \vec{g} = m_g \left(\sum \frac{-K_2 m_g'}{r^2} \hat{r} \right).$$

Here $\vec{g}$ is called the gravitational field acting on m_g due to all the masses m_g'. It is the force per unit gravitational mass.

The electrostatic force between two point charges e and e' is proportional to their product and inversely proportional to the square of their distance r. With a proportionality constant K_3 this can be written as:

$$\vec{F} = K_3 \frac{ee'}{r^2} \hat{r} \ . \tag{3}$$

The force is along the straight line connecting the charges and is repulsive (attractive) if $ee' > 0\,(ee' < 0)$.

The force on a charge e due to several charges e' can be written as

$$\vec{F} = e\vec{E} = e\left(\sum \frac{K_3 e'}{r^2} \hat{r} \right).$$

Here $\vec{E}$ is called the electric field acting on e due to all the charges e'. It is the force per unit charge.

The force between two magnetic poles p and p' separated by a distance r is given by a similar expression:

$$\vec{F} = K_4 \frac{pp'}{r^2} \hat{r}. \tag{4}$$

In the case of long thin bar magnets, the poles are located at the extremities. Usually a north pole of a bar magnet (which points towards the geographic north of the earth) is considered positive and a south pole negative. There will be a force of repulsion (attraction) when $pp' > 0\,(pp' < 0)$. It is also along the straight line connecting the poles.

The force on a magnetic pole p due to several other poles p' can be written as

$$\vec{F} = p\vec{B} = p\left(\sum \frac{K_4 p'}{r^2}\hat{r}\right).$$

Here $\vec{B}$ is called the magnetic field acting on p due to all the poles p'. It is the force per unit magnetic pole.

Between 1820 and 1826 Ampère obtained the force between two current elements. He was led to his researches after Oersted's great discovery of 1820 that a current carrying wire affects a magnet in its vicinity. Following Oersted's discovery, Ampère decided to consider the direct action between currents. From his experiments and theoretical considerations he was led to his force expression. If the circuits carry currents i and i' and the current elements separated by a distance r have lengths ds and ds', respectively, Ampère's force is given by (with a proportionality constant K_5):

$$\begin{aligned} d^2\vec{F} &= K_5 \frac{ii' ds ds'}{r^2}\hat{r}\left(3\cos\theta\cos\theta' - 2\cos\varepsilon\right) \\ &= K_5 \frac{ii'}{r^2}\left[3(\hat{r}\cdot d\vec{s})(\hat{r}\cdot d\vec{s}') - 2(d\vec{s}\cdot d\vec{s}')\right]. \end{aligned} \tag{5}$$

In this expression θ and θ' are the angles between the positive directions of the currents in the elements and the connecting right line between them, ε is the angle between the positive directions of the currents in the elements, $\hat{r}$ is the unit vector connecting them, $d\vec{s}$ and $d\vec{s}'$ are the vectors pointing along the direction of the currents and having magnitude equal to the length of the elements.

After integrating this expression Ampère obtained the force exerted by a closed circuit C' where flows a current i' on a current element $id\vec{s}$ of another circuit as given by:

$$d\vec{F} = id\vec{s} \times \left(K_5 \oint_{C'} \frac{i' d\vec{s}' \times \hat{r}}{r^2}\right).$$

A simple example is given here. Integrating this expression to obtain the force per unit length, dF/ds, due to the interaction between two straight and parallel wires carrying currents i and i' and separated by exerted by a distance ℓ is given by

$$\frac{dF}{ds} = 2K_5 \frac{ii'}{\ell} .$$

This force is attractive (repulsive) if the currents flow in the same (opposite) directions. A modern discussion of Ampère's force between current elements, its integration for different geometries and a comparison with the works of Biot-Savart, Grassmann and Lorentz can be found in Bueno and Assis.[3]

3. Systems of Units

The numerical values and dimensions of the proportionality constants K_1 to K_5 can be chosen arbitrarily. Each choice will influence the numerical values and dimensions of the corresponding physical quantities: inertial mass, gravitational mass, electrical charge, magnetic pole and electric current. The only requirement is that all the forces (1) to (5) have the same dimensions. One possibility, for instance, is to put $K_1 = K_2 = K_3 = K_4 = K_5 = 1$ dimensionless and then adapt the dimensions of m_i, m_g, e, p and i appropriately. Here different options which have been made in the development of physics are discussed.

Combining Eqs. (1) and (2) and analyzing the free fall of a body of constant mass near the surface of the earth (gravitational mass m_{ge} and radius r_e) yields the acceleration of fall as: $a_1 = -(K_2 / K_1)(m_{g1} / m_{i1})(m_{gE} / r_e^2)$. The ratio of the free fall acceleration of body 1 to the free fall acceleration of body 2 at the same spot on the earth's surface is then given by $a_1 / a_2 = (m_{g1} / m_{i1})/(m_{g2} / m_{i2})$. It is an experimental fact discovered by Galileo that two bodies fall freely near the earth's surface with the same acceleration ($a_1 = a_2$), no matter their weight, chemical composition, form etc. This means that the inertial mass of any body is proportional to the gravitational mass of this body, namely: $m_i = K_6 m_g$, where K_6 is a proportionality constant with the same value for all bodies. Combining this with Eq. (2) yields the gravitational force as:

$$\vec{F} = -\frac{K_2}{K_6^2} \frac{m_i m_i'}{r^2} \hat{r} \ . \tag{6}$$

That is, the gravitational force between two bodies is proportional to the product of their inertial masses and inverşely proportional to the square of their distance. Newton presented this law in the *Principia* in terms of these proportionalities.

[3] BUENO and ASSIS (2001).

I discuss now the proportionality constants K_1, K_2 and K_6. The first one of them, K_1, is usually chosen equal to one dimensionless. Supposing a constant mass during the motion this yields Newton's second law in the usual form $\vec{F} = m_i\vec{a}$. Here $\vec{a} = d\vec{v}/dt$ is the acceleration of the body relative to absolute space or to any inertial frame of reference, that is, to any frame of reference which moves with constant velocity relative to absolute space. If the force $\vec{F}$ is constant during the time t, this equations yields $\vec{a} = \vec{F}/m_i =$ constant and $\vec{v} = \vec{v}_o + \vec{a}t$, where $\vec{v}_o$ is the initial velocity of the body.

The unit force is then that constant force which when it acts upon the unit of inertial mass imparts to this mass a unit of velocity in unit of time.[4]

Usually the basic magnitudes of mechanics are chosen to be the inertial mass, length and time; with the other magnitudes (velocities, accelerations, moment etc. based on these 3 magnitudes). Gauss and Weber used to consider milligrams, *mg*, millimeters, *mm*, and seconds, *s*, as their basic magnitudes. In the cgs system they are gram, *g*, centimeter, *cm*, and second, *s*. In the International System of Units MKSA they are kilogram, *kg*, meter, *m*, and second, *s*. Representing these dimensions by $[M_i]$, $[L]$ and $[T]$. With $K_1 = 1$ dimensionless, the dimension of force is then given by $\lfloor M_i LS^{-2} \rfloor$.

Newton estimated the mean density of the earth as between 5 and 6 times the water density. With the measurement of Cavendish for the gravitational force between two globes (utilizing a torsion balance) it was possible to obtain the precise value of the mean density of the earth ($= 5.5\times10^3 kgm^{-3}$). Combining this value with the measurement of the free fall acceleration near the earth's surface and the value of its radius, it is possible to obtain from Eq. (6) the value of $K_2/K_6^2 = 6.67\times10^{-11} kg^{-1}m^3s^{-2}$. Usually this is represented by *G*, called the gravitational constant.

In one system of units $K_1 = K_2 = 1$ dimensionless. The unit of gravitational mass is then defined as the mass which acting on another equal unit gravitational mass separated by a unit of distance generates a unit force. In this case: $m_g = \sqrt{G}m_i$.[5]

In another system of units, the so-called astronomical system, $K_1 = K_2/K_6^2 = 1$ dimensionless. In this case the dimension of inertial mass is given by $[L^3S^{-2}]$ and is

[4] WEBER (1872), especially p. 2.
[5] PALACIOS (1964).

not considered any more an independent magnitude, as it can be deduced or derived from the dimensions of length and time.

The first system of units applicable to electric quantities to be considered here is the electrostatic. In this system $K_3 = 1$ dimensionless and the dimension of the charges e and e' is called electrostatic unit, *esu*. Two equal charges $e = e'$ are said to have unit magnitude when they exert upon one another a unit force when separated by a unit distance.

The second system of units utilized during the XIXth century is the electromagnetic system of units. In it $K_4 = 1$ dimensionless and the dimension of the magnetic poles p and p' is called electromagnetic unit, *emu*. Once more two equal magnetic poles $p = p'$ are said to have unit magnitude when exert a unit of force when separated by a unit distance. Gauss in 1832 was the first to introduce this system of units with $K_4 = 1$.[6]

For a biography of Gauss with many references, see Reich.[7]

The physical connection between magnetic pole and current was given by Oersted's experiment of 1820. That is, he observed that a galvanic current orients a small magnet in the same way as others magnets (or the earth) do.

From Ampère's force law it is possible to obtain a mathematical connection between these two concepts. This is done writing the integrated expression of Ampère's force as

$$d\vec{F} = id\vec{s} \times \vec{B},$$

where $\vec{B}$ is called the magnetic field generated by the closed circuit C'. It is only possible to call it a magnetic field by Oersted's experiment. That is, the force exerted on a unit magnetic pole located at the same place as $id\vec{s}$ by the current carrying circuit C' is given by this magnetic field. This means that p and ids have the same units.

Comparing the magnetic field of this equation with that given by magnetic poles yields

$$K_4 = K_5.$$

Alternatively it is possible to compare a magnetic pole and a galvanic current (or connect the constants K_4 and K_5) considering the known fact described by Maxwell in the following words:

[6] GAUSS (1832).
[7] REICH (1977).

> It has been shown by numerous experiments, of which the earliest are those of Ampère, and the most accurate those of Weber, that the magnetic action of a small plane circuit at distances which are great compared with the dimensions of the circuit is the same as that of a magnet whose axis is normal to the plane of the circuit, and whose magnetic moment is equal to the area of the circuit multiplied by the strength of the current.[8]

The expression *magnetic action* can be understood here as the force or torque of the small circuit or of the small magnet acting on another small magnet. It is also possible to say that the magnetic field exerted by this small circuit is the same as that generated by the small magnet, provided that

$$p\ell\hat{\ell} = iA\hat{u}.$$

Here i is the current of the small plane circuit of area A and normal unit vector $\hat{u}$, p is the magnetic pole of the small magnet of length ℓ and $\hat{\ell}$ points from the south to the north pole, $p\ell\hat{\ell} = p\vec{\ell}$ being the magnetic moment of the magnet. As ℓ has the unit of length and A has the unit of length squared, the ratio of p/i has the unit of length.

Ampère, who obtained for the first time a mathematical expression for the force between current-carrying circuits utilized what is called the electrodynamic system of units. In this system $K_4 = K_5 = 1/2$ dimensionless and the currents are measured in (or its units and dimensions are) electrodynamic units. On the other hand, in the electromagnetic system $K_4 = K_5 = 1$ dimensionless and the currents are measured in electromagnetic units.[9]

The electrodynamic system of units was adopted by Ampère but has since been abandoned. In any event it is relevant to compare the currents in electrodynamic and in electromagnetic measures. The strengths of the currents in electrodynamic measure can be represented by j and j', and the same currents in electromagnetic measure can be represented by i and i'. By the fact that $K_5 = 1$ in the electromagnetic system and that $K_5 = 1/2$ in the electrodynamic system the following relation is obtained: $jj'/2 = ii'$ or $j = \sqrt{2}\,i$, if there is the same current in both wires (i = i' and j = j'). In order to compare the unit current in electromagnetic measure with the unit current in electrodynamic measure, it is convenient to consider the previous example of two parallel wires carrying the same current. The force per unit length (*dF/ds'*) between them if they are separated by a unit distance is given by 2 force units per length unit if i = i' = 1 unit

[8] MAXWELL (1954), article 482, p. 141.

[9] TRICKER (1965), pp. 25, 51, 56 and 73.

electromagnetic current, remembering that $K_5 = 1$ in electromagnetic measure. On the other hand, if $j = j' = 1$ unit electrodynamic current, $dF/ds' = 1$ force unit per length unit, if they are separated by a unit distance, remembering that $K_5 = 1/2$ in electrodynamic measure. This means that in order to generate the same effect as one electromagnetic unit of current (that is, to have the same force between the wires), it is necessary to have $\sqrt{2}$ electrodynamic units of current. Hence the unit current adopted in electromagnetic measure is greater than that adopted in electrodynamic measure in the ratio of $\sqrt{2}$ to 1.[10,11] That is, although $j = \sqrt{2}\, i$, a unit electromagnetic unit of current is equal to (has the same effect of, or generates the same force of) $\sqrt{2}$ units of electrodynamic current.

The connection between the electric currents (or between the units of charge) in electrostatic and in electromagnetic units is considered below.

In the International System of Units MKSA the basic dimensions for *length, mass, time* and electric current are given by meter (m), kilogram (kg), second (s) and Ampère (A). Forces are expressed in the dimension Newton ($1N = 1kgms^{-2}$) and electric charges in Coulomb ($1C = 1As$). In this system the constants discussed in this work are given by: $K_1 = 1$ dimensionless and $K_2/K_6^2 = G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$. Moreover, $K_3 = 1/(4\pi\varepsilon_o)$, where $\varepsilon_o = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$ is called the permittivity of free space. The constant $K_4 = K_5 = \mu_o/(4\pi)$, where μ_o is called the vacuum permeability. By definition its value is given by $\mu_o = 4\pi \times 10^{-7} kgmC^{-2}$. In this case the dimensions of the magnetic poles p and p' are $Am = Cm/s$. The constant c is related with ε_o and μ_o by $c = 1/\sqrt{\mu_o \varepsilon_o}$. Of these three constants (ε_o, μ_o and c), only one is measured experimentally, c. The value of μ_o is given by definition, with ε_o is obtained by $\varepsilon_o = 1/(c^2 \mu_o)$.

4. Weber's Electrodynamics

The fundamental law is now discussed describing the interaction between charges formulated by Wilhelm Weber (1804-1891). Weber's complete works can be found

[10] MAXWELL (1954), article 526, p. 173.
[11] TRICKER (1965), p. 51.

in: Weber (1892-94).[12] For a biography of Weber see Wiederkehr.[13] A modern discussion of Weber's force applied to electromagnetism and gravitation, with which it is possible to implement Mach's principle, with many references to be found in Assis[14,15] and Bueno and Assis.[3]

In order to unify electrostatics (Coulomb's force of 1785) with electrodynamics (Ampère's force between current elements of 1826) and with Faraday's law of induction (1831), Wilhelm Weber proposed in 1846 the following force between two point charges *e* and *e*' separated by a distance *r*:

$$\vec{F} = K_3 \frac{ee'}{r^2}\hat{r}\left(1 - \frac{a^2}{16}\dot{r}^2 + \frac{a^2 r\ddot{r}}{8}\right),$$

In this equation $\dot{r} = dr/dt$, $\ddot{r} = d^2r/dt^2$ and *a* is a constant which Weber only determined 10 years later. The charges *e* and *e*' may be considered as localized at $\vec{r}_1$ and $\vec{r}_2$ relative to the origin *O* of an inertial frame of reference *S*, with velocities and accelerations given by, respectively, $\vec{v}_1 = d\vec{r}_1/dt$, $\vec{v}_2 = d\vec{r}_2/dt$, $\vec{a}_1 = d\vec{v}_1/dt$ and $\vec{a}_2 = d\vec{v}_2/dt$. The unit vector pointing from 2 to 1 is given by $\hat{r} = (\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|$. In this way $r = |\vec{r}_1 - \vec{r}_2| = \sqrt{(\vec{r}_1 - \vec{r}_2)\cdot(\vec{r}_1 - \vec{r}_2)}$, $\dot{r} = \hat{r}\cdot(\vec{v}_1 - \vec{v}_2)$ and $\ddot{r} = [(\vec{v}_1 - \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2) - (\hat{r}\cdot(\vec{v}_1 - \vec{v}_2))^2 + (\vec{r}_1 - \vec{r}_2)\cdot(\vec{a}_1 - \vec{a}_2)]/r$. Weber wrote this equation with $K_3 = 1$ dimensionless and without vectorial notation.

By 1856 Weber was writing this equation with *c* instead of *4/a*. But Weber's *c* = *4/a* is not the present day value $c = 3\times10^8\, m/s$, but $\sqrt{2}$ this last quantity. To avoid confusion with the modern *c*, and following the procedure adopted by Rosenfeld.[16] Weber's *4/a* will be represented here by c_W. This means that by 1856 Weber was writing his force law as the middle term below (the term on the right hand side is the modern rendering of Weber's force with the present day value of *c*):

$$F = K_3 \frac{ee'}{r^2}\left(1 - \frac{1}{c_W^2}\dot{r}^2 + \frac{r\ddot{r}}{2c_W^2}\right) = K_3 \frac{ee'}{r^2}\left(1 - \frac{1}{2c^2}\dot{r}^2 + \frac{r\ddot{r}}{c^2}\right).$$

[12] WEBER (1892-4).
[13] WIEDERKEHR (1967).
[14] ASSIS (1994).
[15] ASSIS (1999a).
[16] ROSENFEL (1957).

If there is no motion between the point charges, $\dot{r}=0$ and $\ddot{r}=0$, Weber's law reduces to Coulomb's force. This means that the whole of electrostatics (Gauss's law etc.) are embodied in Weber's electrodynamics.

Weber knew in 1846 Coulomb's force between point charges and Ampère's force between current elements. He arrived at his force from these two expressions and a connection between current and charges. A description of his procedure can be found in his work and also in Maxwell and Whittaker's books:Weber,[17] Maxwell[18] and Whittaker.[19] Here the opposite approach is followed, namely, beginning with Weber's force in order to arrive at Ampère's force.

Consider then the force between two current elements, 1 and 2. The positive and negative charges of the first one are represented by de_{1+} and de_{1-}, while those of element 2 are de_{2+} and de_{2-}. Supposing that they are electrically neutral yields $de_{1-}=-de_{1+}$ and $de_{2-}=-de_{2+}$. As a matter of fact there is always some net charge inside and along the surface of resistive wires, but the effects produced by these charges are usually small,[20] which means that this is a reasonable approximation. Adding Weber's force exerted by the positive and negative charges of the neutral element 1 on the positive and negative charges of the neutral element 2 yields:[21]

$$F=K_3\frac{de_{1+}de_{2+}}{r^2}\frac{1}{c^2}\left\{3[\hat{r}\cdot(\vec{v}_{1+}-\vec{v}_{1-})][\hat{r}\cdot(\vec{v}_{2+}-\vec{v}_{2-})]-2(\vec{v}_{1+}-\vec{v}_{1-})\cdot(\vec{v}_{2+}-\vec{v}_{2-})\right\}.$$

In order to arrive at Ampère's force from this expression a relation between current and charge is necessary. The commonly accepted definition of current is the time rate of change of charge, that is, a current is the amount of charge transferred through the cross section of a conductor per unit time:

$$i=\frac{de}{dt}.$$

[17] WEBER (1966).
[18] MAXWELL (1954), chapter XXIII.
[19] WHITTAKER (1973), pp. 201-3.
[20] ASSIS, RODRIGUES, and MANIA (1999).
[21] ASSIS (1994), section 4.2.

If the charge is measured or expressed in electrostatic, electromagnetic or electrodynamic units, the current will also be measured or expressed in electrostatic, electromagnetic or electrodynamic units, respectively.[22]

Applying this definition in Ampère's expression for the force between current elements, Eq. (5), and comparing it with Eq. (3) yields a relation between the dimensions of K_3 and K_5. That is, the ratio K_3 / K_5 has the unit of a velocity squared. It is independent of the units of electric and magnetic quantities and is a fundamental constant of nature.

Fechner and Weber supposed in 1845-46 that galvanic currents consist of an equal amount of positive and negative charges moving in opposite directions with the same velocity relative to the wire.[23] Nowadays it is known that the usual currents in metallic conductors are due to the motion of only the negative electrons. But it is possible to derive Ampère's force from Weber's one even without assuming Fechner's hypothesis, (Wesley,[24] Assis[25,26]).

Utilizing $i = de/dt$ and $\vec{v} = d\vec{s} / dt$ in the expression for the force between current elements yields

$$d^2F = \frac{K_3}{c^2}\frac{ii'}{r^2}\left[3(\hat{r}\cdot d\vec{s})(\hat{r}\cdot d\vec{s}') - 2(d\vec{s}\cdot d\vec{s}')\right]$$

This will be Ampère's force provided $K_3 / c^2 = K_5$, that is:

$$c = \sqrt{\frac{K_3}{K_5}}.$$

As has been said before, integrating Ampère's expression for the force exerted by an infinitely long straight wire carrying a constant i' acting on a current element ids parallel and at a distance ℓ to it is given by

$$dF = 2\frac{K_3}{c^2}\frac{ii'ds'}{\ell}.$$

[22] MAXWELL (1954), articles 231, 626 and 771.
[23] WHITTAKER (1973), p. 201.
[24] WESLEY (1990).
[25] ASSIS (1990).
[26] ASSIS (1994), section 4.2.

Utilizing electrostatic units ($K_3 = 1$ dimensionless), the force per unit length (*dF/ds'*) between them if they are separated by a unit distance is given by $2/c^2$ force units per length unit if *i* = *i'* = 1 electrostatic unit. On the other hand it was shown above that in electromagnetic units if *i* = *i'* = 1 electromagnetic unit than *dF/ds'* will be given by 2. For the current in electrostatic units generate the same force per unit length its magnitude needs to be given by *c* units. This means that *c* is the ratio of electromagnetic and electrostatic units of current, or the ratio of electromagnetic and electrostatic units of charge.

For this reason it is possible to write

$$de_{\text{electromagnetic measure}} = \frac{de_{\text{electrostatic measure}}}{c}.$$

Alternatively it might also be said that *c* is the number of units of static electricity which are transmitted by the unit electric current in the unit of time. That is, if two equal unit electrostatic charges are separated by a unit distance, they exert a unit force on each other according to Eq. (3). By combining this last equation with Eq. (3) it is possible to write $F = c^2 ee'/r^2$, where *e* and *e'* are the charges in electromagnetic units ($K_3 = c^2$ in electromagnetic measure). If two equal unit electromagnetic charges are separated by a unit distance they exert on each other a force of magnitude c^2 units of force. In order to generate a unit force (as two unit electrostatic forces do), it is necessary to have *e* = *e'* = *c* electromagnetic units. Analogously the constant $c_W = \sqrt{2}\, c$ is the ratio of the electrodynamic and electrostatic units of charge.

Charges are usually obtained in electrostatic units, measuring directly the force between charged bodies. Currents, on the other hand, are usually obtained in electromagnetic units. That is, the force is measured between current carrying circuits or the deflection of a galvanometer (torque due to the forces between current carrying conductors). Alternatively it can be measured the torque or deflection of a small magnet due to a current carrying wire. But in order to know the numerical value of K_3 / K_5 it is necessary to measure electrostatically the force between two charged bodies, discharge them and measure this current electromagnetically. Then it will be possible to express currents (and charges) measured in electromagnetic units in terms of currents (and charges) expressed in electrostatic units.

The first measurement of c_W was performed by Weber and Kohlrausch in 1855, when there was the first public announcement of its value.[27] The complete paper

[27] WEBER (1855).

was published in 1857.[28] An abstract of this paper appeared 1956 in Weber and Kohlrausch,[29] with English translation in 1996.[30] Weber and Kohlrausch found $c_W = \sqrt{2}\, c = 4.39 \times 10^8\, m/s$, such that $c = 3.1 \times 10^8\, m/s$. This was one of the first quantitative measurements indicating a possible connection between electromagnetism and optics. Discussions of this measurement can be found in: Kirchner,[31] Wiederkehr[32,39] Woodruff[33,35] Rosenfeld[34,15] Wise,[36] Harman,[37] Jungnickel and McCormmach,[38] and D'Agostino.[40]

5. Propagation of Electromagnetic Signals

The first to derive the correct equations describing the propagation of electromagnetic signals in wires (telegraphy equation) were Weber and Kirchhoff in 1857, before the works of Maxwell and Heaviside. Kirchhoff worked with Weber's action at a distance theory and has three main papers related directly with this, one of 1850 and two of 1857, all of them have been translated to English.[41,42,43] Weber's simultaneous and more thorough work was delayed in publication and appeared only in 1864.[44] Both worked independently of one another and predicted the existence of periodic modes of oscillation of the electric current propagating at light velocity in a conducting circuit of negligible resistance.

A discussion of the procedure followed by Kirchhoff in modern notation utilizing the International System of Units MKSA has been given in Assis.[45,1] It is presented here once more for the sake of completeness. In Assis[46] this approach was

[28] KOHLRAUSCH and WEBER (1857).
[29] WEBER and KOHLRAUSCH (1956).
[30] WEBER and KOHLRAUSCH (1996).
[31] KIRCHNER (1957).
[32] WIEDERKEHR (1967), pp. 138-41.
[39] WIEDERKEHR (1994).
[33] WOODRUFF (1968).
[35] WOODRUFF (1976).
[34] ROSENFELD (1973).
[36] WISE (1981).
[37] HARMAN (1982).
[38] JUNGNICKEL and MCCORMMACH (1986), pp. 144-6 and 296-7.
[40] D'AGOSTINO (1996).
[41] KIRCHHOFF (1950).
[42] KIRCHHOFF (1957).
[43] GRANEAU and ASSIS (1994).
[44] WEBER (1864).
[45] ASSIS (1999b).
[46] ASSIS (2000b).

applied to the case of coaxial cables, which had not been considered by Kirchhoff and Weber.

In his first paper of 1857, Kirchhoff considered a conducting circuit of circular cross section which might be open or closed in a generic form. He wrote Ohm's law taking into account the free electricity along the surface of the wire and the induction due to the alteration of the value of the current in all parts of the wire, $\vec{J} = -g\left(\nabla\phi + \frac{\partial \vec{A}}{\partial t}\right)$.

Here $\vec{J}$ is the current density, g the conductivity of the wire, ϕ is the electric potential and $\bar{A}$ the magnetic vector potential. He calculated ϕ integrating the effect of all surface free charges, $\phi(x,y,z,t) = \frac{1}{4\pi\varepsilon_o}\iint\frac{\sigma(x',y',z',t)da'}{|\vec{r}-\vec{r}'|}$. Here $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the point where the potential is being calculated, t is the time and σ is the surface density of charges. After integrating over the whole surface of the wire of length ℓ and radius α he arrived at $\phi(s,t) = \frac{\alpha\,\sigma(s,t)}{\varepsilon_o}\ln\frac{\ell}{\alpha}$, where s is a variable distance along the wire from a fixed origin. The vector potential $\bar{A}$ he obtained from Weber's formula as given by

$\bar{A}(x,y,z,t) = \frac{\mu_o}{4\pi}\iiint[\bar{J}(x',y',z',t)\cdot(\vec{r}-\vec{r}')](\vec{r}-\vec{r}')\frac{dx'dy'dz'}{|\vec{r}-\vec{r}'|^5}$. Here the integration is through the volume of the wire. After integrating this expression he arrived at $\vec{A}(s,t) = \frac{\mu_o}{2\pi}I(s,t)\ln\frac{\ell}{\alpha}\hat{s}$, where $I(s,\ t)$ is the variable current. Considering that $I = J\pi\alpha^2$ and that $R = \ell/(\pi g\alpha^2)$ is the resistance of the wire, the longitudinal component of Ohm's law could then be written as $\frac{\partial\sigma}{\partial t} + \frac{1}{2\pi\alpha}\frac{1}{c^2}\frac{\partial I}{\partial t} = -\frac{\varepsilon_o R}{\alpha\,\ell\ln(\ell/\alpha)}I$. In order to relate the two unknowns σ and I Kirchhoff utilized the equation for the conservation of charges which he wrote as $\frac{\partial I}{\partial t} = -2\pi\alpha\frac{\partial\sigma}{\partial t}$. By equating these two relations it is obtained the equation of telegraphy, namely:

$$\frac{\partial^2\xi}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2\xi}{\partial t^2} = \frac{2\pi\varepsilon_o R}{\ell\ln(\ell/\alpha)}\frac{\partial\xi}{\partial t},$$

where ξ can represent I, σ, ϕ or the longitudinal component of $\vec{A}$. If the resistance is negligible, this equation predicts the propagation of signals along the wire with light velocity.

Although in this derivation the interaction between any two charges is given by Weber's action at a distance law, the collective behavior of the disturbance propagates at light velocity along the wire. This is somewhat similar to the propagation of sound waves derived by Newton or the propagation of signals along a stretched string obtained by d'Alembert. In all these cases classical Newtonian mechanics was employed, without time retardation, without displacement current and without any field propagating at a finite speed. Although the interaction of any two particles in all these cases was of the type action at a distance, the collective behavior of the signal or disturbance did travel at a finite speed.

In these cases there is a many-body system (molecules in the air, molecules in the string or charges in the wire) in which the particles had inertia. Is it possible to derive the propagation of electromagnetic signals in vacuum, as in radio communication, by an action at a distance theory? I believe the answer to this question is positive. In practice there is never only a two-body system. In any antenna there are many charged particles. Even if the material medium (like air) between two antennae is removed, there is always a gas of photons in the space between them. It is possible that each photon be like an electric dipole, with the opposite charges oscillating or vibrating, while at the same time the photon as a whole moves with light velocity. The action at a distance between the charges in both antennae with one another and with the gas of photons in the intervening space may give rise to a collective behavior which is called electromagnetic radiation propagating at light velocity. Moreover, by Mach's principle the distant universe must always be taken into account. After all, the inertial properties of any charge is due to its gravitational interaction with the distant matter in the cosmos.[15] For this reason there is always a many body interaction in any real situation. This means that there may be expected the derivation of the propagation of electromagnetic signals in vacuum moving at light velocity, supposing only Weber's action at a distance force law, by analogy with what Kirchhoff and Weber accomplished in the case of telegraphy.

6. Conclusion and Discussion

The constant c (or $c_W = \sqrt{2}\,c$) was introduced in electromagnetic theory by Weber in 1846. His goal was to unify electrostatics (Coulomb's force) with electrodynamics (Ampère's force) in a single force law. It is the ratio of electromagnetic (or electrodynamic) and electrostatic units of charge. Weber was also the first to measure this quantity working together with Kohlrausch. Their work is from 1855 and they obtained $c = 3.1\times10^8\,m/s$ (or $c_W = 4.4\times10^8\,m/s$). Weber and Kirchhoff were also the first to obtain the equation of telegraphy describing the propagation of electromagnetic signals along wires. In the case of negligible resistance they obtained the wave equation with a characteristic velocity given by c. These were some of the first connections between electromagnetism and optics as the value of light velocity was known to be $3\times10^8\,m/s$, the same value obtained for c by Weber and Kohlrausch's experiment.

It should be mentioned that one of the meanings which Weber gave to the constant c_W was that of a limiting velocity. That is, according to Weber's force if two charges are approaching or moving away from one another with a constant relative radial velocity $\dot{r} = \pm c_W$, such that $\ddot{r} = 0$, then the net force between them would be zero.

The electrostatic force would be cancelled by the component of the force which depends on the relative velocity and they would move with constant velocities (if they were not interacting with other bodies), as if the other charge did not exist. It seems to me that Weber was one of the first to speak of a limit velocity in physics connected with a dynamical force law.

It should be stressed that the works of Weber and Kirchhoff in 1856-57 were performed before Maxwell wrote down his equations in 1864. When Maxwell introduced the displacement current $(1/c^2)\partial\vec{E}/\partial t$ he was utilizing Weber's constant c. He was also aware of Weber and Kohlrausch's measurement of 1855 that c had the same value as light velocity. He also knew Weber and Kirchhoff's derivation of the telegraphy equation yielding the propagation of electromagnetic signals at light velocity.

For detailed work describing the link between Weber's electrodynamics and Maxwell's electromagnetic theory of light the following works are recommended: Wiederkehr[39] and D'Agostino.[40]

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Appendix

Wilhelm Weber and Rudolf Kohlrausch

On the Amount of Electricity which Flows through the Cross-Section of the Circuit in Galvanic Currents*)

[Translated by Susan P. Johnson and edited by Laurence Hecht]

A prefatory note from Kohlrausch says that the publisher desired for the Annals a report on work carried out jointly by Weber and Kohlrausch, whose results were presented in a more fundamental and conclusive way by Weber in vol. V of the treatises of the Royal Saxon Scientific Society in Leipzig, under the title *Elektrodynamische Maassbestimmungen, insbesondere Zurueckfuehrung der Stroemintensitaetsmessungen auf mechanisches Maass*, Leipzig, S. Hirzel, 1856. "Herewith I give a short precis".

1. Problem

The comparison of the effects of a closed galvanic circuit with the effects of the discharge-current of a collection of free electricity, has led to the assumption, that these effects proceed from a *movement of electricity* in the circuit. We imagine that in the bodies constituting the circuit, their *neutral* electricity is in motion, in the manner that their entire positive component pushes around in the one direction in closed, continuous circles, the negative in the opposite direction. The fact that an accumulation of electricity never occurs by means of this motion, requires the assumption, that the same amount of electricity flows through each cross-section in the same time-interval.

It has been found suitable to make the *magnitude of the flow*, the so-called *current intensity*, proportional to the amount of electricity which goes through the cross-section of the circuit in the same time-interval. If, therefore, a certain current intensity is to be expressed by a *number*, it must be stated, which current intensity is to serve as the measure, i.e., which magnitude of flow will be designated as 1.

*) *Poggendorf's Annalen*, vol. XCIX, pp. 10-25.

Here it would be simplest, as in general regarding such flows, to designate as 1 that magnitude of flow which arises, when in the time-unit the unit of flow goes through the cross-section, thus defining the measure of current intensity from its *cause*. The unit of electrical fluid is determined in electrostatics by means of the *force*, with which the free electricities act on each other at a distance. If one imagines two equal amounts of electricity of the same kind concentrated at two points, whose distance is the unit of length, and if the force with which they act on each other repulsively, is equal to the *unit of force*, then the amount of electricity found in each of the two points is the measure or the unit of free electricity.

In so doing, *that force* is assumed as the unit of force, *through which the unit of mass is accelerated around the unit of length during the unit of time*. According to the principles of mechanics, by establishing the units of length, time, and mass, the measure for the force is therefore given, and by joining to the latter the measure for free electricity, we have at the same time a measure for the current intensity.

This measure, which will be called the *mechanical measure* of current intensity, thus *sets as the unit, the intensity of those currents which arise when, in the unit of time, the unit of free positive electricity flows in the one direction, an equal amount of negative electricity in the opposite direction, through that cross-section of the circuit.*

Now, according to this measure, we cannot carry out the measurement of an existing current, for we know neither the amount of neutral electrical fluid which is present in the cubic unit of the conductor, nor the velocity, with which the two electricities displace themselves [sich verschieben] in the current. We can only compare the intensity of the currents by means of the *effects* which they produce.

One of these effects is, e.g., the *decomposition of water*. Sufficient grounds converge, to make the current intensity proportional to the amount of water, which is decomposed in the same time-interval. Accordingly, *that current intensity will be designated as 1, at which the mass-unit of water is decomposed in the time-unit*, thus, e.g., if seconds and milligrams are taken as the measure of time and mass, that current intensity, at which in one second one milligram of water is decomposed. *This measure of current intensity is called the electrolytic measure.*

The natural question now arises, how this electrolytic measure of current intensity is related to the previously established mechanical measure, thus the question, how many (electrostatically or mechanically measured) positive units of electricity flow through the cross-section in one second, if a milligram of water is decomposed in this interval of time.

Another effect of the current is the *rotational moment* it exerts on a magnetic needle, and which we likewise assume to be proportional to the current intensity, conditions being otherwise equal. If a current intensity is to be measured by means of this kind of effect, then the *conditions* must be established, under which the rotational moment is to be observed. One could designate as 1 that current intensity which under *arbitrarily* established spatial conditions exerts an *arbitrarily* established rotational moment on an arbitrarily chosen magnet. When, then, under *the same* conditions, an *m*-fold large rotational moment is observed, the current intensity prevailing in this case would have to be designated as m. Precisely the impracticability of such an arbitrary measure, however, has led to the *absolute* measure, and thus in this case the electromagnetic measure of current intensity is to be joined to the absolute measure for magnetism. This occurs by means of the following specification of *normal conditions* for the observation of the magnetic effects of a current:

The current goes through a circular conductor, which circumscribes the unit of area, and acts on a magnet, which possesses the unit of magnetism, at an arbitrary but large distance = R; the midpoint [center] of the magnet lies in the plane of the conductor, and its magnetic axis is directed toward the center of the circular conductor. – The rotational moment D, exerted by the current on the magnet, expressed according to mechanical measure, is, under these conditions, different according to the difference in the current intensity, and also according to the difference in the distance R; the product R^3D *depends, however, simply on the current intensity*, and is hence, under these conditions, the *measurable* effect of the current, namely, that effect by means of which the current intensity is to be measured, according to which one therefore obtains as *magnetic measure of current intensity* the intensity of that current, for which $R^3D = 1$. – The electromagnetic laws state, that this measure of current intensity is also the intensity of that current which, if it circumscribes a plane of the size of the unit of area, everywhere exerts at a distance the effects of a magnet located at the center of that plane, which possesses the unit of magnetism and whose magnetic axis is perpendicular to the plane; – or also, that it is the intensity of that current, by which a *tangent boussole with simple rings of radius* = R is kept in equilibrium, given a deflection from the magnetic meridian

$$\varphi = \arctan \frac{2\pi}{RT}$$

if T denotes the horizontal intensity of the terrestrial magnetism.

Here, too, arises the natural question about the relation of the *mechanical* measure of current intensity to this *magnetic* measure, thus the question, how many times the electrostatic unit of the volume of electricity must go through the cross-section of the circuit during one second, in order to elicit that current intensity, of which the just-specified deflection, φ, is effected by the needle of a tangent boussole.

The same question, repeats itself in considering a *third* measure of current intensity, which is derived from the electrodynamic effects of the current, and is therefore called the *electrodynamic measure* of the current intensity.

The three measures drawn from the *effect* of the currents have already been compared with one another. It is known that the magnetic measure is $\sqrt{2}$ larger than the electrodynamic, but $106\frac{2}{3}$ times smaller than the electrolytic, and for that reason, in order to solve the question of how these three measures relate to *mechanical* measure, it is merely necessary to compare the later with one of the others.

This was the goal of the work undertaken, which goal was to be attained through the solution of the following problem:

Given a constant current, by which a tangent boussole with a simple multiplier circle or radius = R^{mm} is kept in equilibrium at a deflection $\varphi = \arctan\frac{2\pi}{RT}$ *if T is the intensity of the horizontal terrestrial magnetism affecting the boussole: Determine the amount of electricity, which flows in such a current in one second through the cross-section of the conductor, relates to the amount of electricity on each of two equally charged (infinitesimally) small balls, which repel one another at a distance of 1 millimeter with the unit of force. The unit of force is taken as that force, which imparts 1 millimeter velocity to the mass of 1 milligram in 1 second.*

2. Solution of this Problem

If a volume *E* of free electricity is collected at an insulated conductor and allowed (by inserting a column of water) to flow to earth through a multiplier, the magnetic needle will be deflected. The magnitude of the first deflection depends, given the same multiplier and the same needle, solely on the amount of discharged electricity, since the discharge time is so short, compared with the oscillation period of the needle, that the effect must be considered as an impulse.

If a constant current is put through a multiplier for a similarly short time, the needle receives a similar impulse, and in this case as well, the magnitude of the first deflection depends *solely on the amount of electricity* which moves through the cross-section of the multiplier wire during the duration of the current.

Now, if in the same multiplier, *exactly the same* deflection were to occur, the one time, when the known amount of free electricity *E* was discharged, the other time, when one let a *constant current* act briefly, then, as can be proven, the amount of positive electricity, which flows during this short time-interval in the constant current, in the direction of this current, through the cross-section, equals *E*/2.

Accordingly, the problem posed requires the solution of the following two problems:

a) measuring the collected amount E of free electricity with the given electrostatic measure, and observing the deflection of the magnetic needle when the electricity is discharged;

b) determining the small time-interval τ, during which a constant current of intensity = 1 (according to magnetic measure) has to flow through the multiplier of the same galvanometer, in order to impart to the needle the same deflection.

If next we multiply $E/2$ by the number which shows how often τ is contained in the second, then the number $\frac{E}{2\tau}$ expresses the amount of positive electricity, which, in a current whose intensity = 1 according to magnetic measure, passes through the cross-section of the conductor in the direction of the positive current in 1 second.

Problem a is treated in the following way:

First, with the help of the sine-electrometer, the conditions are determined with greater precision, in which the charge of a small Leyden jar is divided between the jar itself and an approximately 13-inch ball coated with tin foil, which was suspended, by a good insulator, away from the walls of the room, so that from the amount of electricity flowing on the ball, as soon as it was able to be measured, the amount remaining in the little jar could also be calculated down to a fraction of a percent.

The observation consisted of the following:

The jar was charged, the large ball put in contact with its knob; three seconds later, the charge remaining in the jar was discharged through a multiplier[1] consisting of 5635 windings, by the insertion of two long tubes filled with water, and the first deflection φ of the magnetic needle, which was equipped with a mirror in the manner of the magnetometer, was observed. At the same time, the large ball was now put in contact with the approximately 1-inch fixed ball of a torsion balance[2] constructed on a very large scale. This fixed ball, brought to the torsion balance, shared its received charge with [or: gave half its received charge to] the moveable ball, which made it possible to measure the torsion which was required, to a decreasing extent over time, in order to maintain the two balls at a fully determinate, pre-ascertained distance. – From the torsion coefficients of the wire, found in the manner well known from oscillation experiments, and the precisely determined dimensions, the amount of electricity occurring at each moment in the torsion balance could be measured in the required absolute measure, taking into consideration the non-uniform distribution of electricity in the two balls (which consideration was advisable because of the not insignificant size of the balls compared with the distance between them). The observed decrease in torsion also

[1] The mean diameter of the windings was 266 mm; the almost 2/3-mile-long wire, very well coated with silk, was previously drawn through collodium along its entire length, while the sides of the casing were strongly coated with sealing wax. A powerful copper damper moderated the oscillations.

[2] The frame of the torsion balance, in whose center the balls were located, was in the shape of a parallelepiped 1.16 meters long, 0.81 meters wide, and 1.44 meters high. The long shellac pole [Stange], to which the moveable bass was affixed by means of a shellac side-arm, allowed the observation of the position of the ball under a mirror, and then dipped into a container of oil, by means of which the oscillations were very quickly halted.

yielded the loss of electricity, so that it was possible, by means of this consideration, to state how large these amounts would be, if they could already have been in the torsion balance at the moment at which the large ball was charged by the Leyden jar. From the precisely measured diameter of these balls, the proportion of the distribution of electricity between them could be determined (according to Plana's work), so that, by means of the measurement in the torsion balance, without further ado, it was known what amount of electricity remained in the Leyden jar after charging the large ball, and what amount was discharged 3 seconds later by the multiplier. Only one small correction was still required on account of the loss of available discharge, which occurred during these 3 seconds from leakage into the air and through residue formation.

In the following table are assembled the results of five successive experiments. The column headed E contains the amounts of discharged electricity, the column headed s the corresponding deflections of the magnetic needle in scale units, and the column headed φ the same deflections, but in arcs for radius = 1.

N°	E	s	φ
1	36060000	73.5	0.0057087
2	41940000	80.0	0.0062136
3	49700000	96.5	0.0074952
4	44350000	91.1	0.0070757
5	49660000	97.8	0.0075962

Problem b requires knowing the time-intervals τ, during which a current of that intensity denoted 1 in magnetic current measure, must flow through the same multiplier, in order to elicit the deflections φ observed in the five experiments.

The rotational moment, which is exerted by the just-designated currents on a magnetic needle, which is parallel to the windings of the multiplier, is developed in the *second part of the* Electrodynamische Maassbestimmungen *of W. Weber*. This rotational moment is proportional to the magnetic moment of the needle and the number of windings, but moreover is a function of the dimensions of the multiplier and the distribution of magnetic fluids in the needle, for which it suffices, to determine the distance of the centers of gravity of the two magnetic fluids, which, in lieu of the actual distribution of magnetism, can be thought of as distributed on the surface of the needle. The needle always remaining small compared with the diameter of the multiplier, for this distance a value derived from the size of the needle could be posited with sufficient reliability, so that the designated rotational moment D contains only the magnetic moment of the needle as an unknown. – If this rotational moment acts during a time-interval τ, which is very short compared with the oscillation period of the needle, then the angular velocity imparted to the needle is expressed by

$$\frac{E}{K}\tau,$$

where K signifies the inertial moment. The relationship between this angular velocity and the first deflection φ then leads to an equation between τ and φ,

$$\tau = \varphi A,$$

in which A consists of magnitudes to be truly rigorously measured, thus signifies known constants, namely $A = 0.020915$ for the second as measure of time.

Thus, if it is asked how long a time-interval τ a constant current of magnetic current intensity = 1 has to flow through the multiplier, in order to elicit the above-cited five observed deflections, one need only insert their values for τ into this equation. In this way the values in seconds result as

N°	τ
1	0.0001194
2	0.0001300
3	0.0001568
4	0.0001480
5	0.0001589

If we now divide $E/2$ in the five experiments by the pertinent τ, we obtain

N°	$\frac{E}{2\tau}$
1	151000×10^6
2	161300×10^6
3	158500×10^6
4	149800×10^6
5	156250×10^6

thus as a mean,

$$\frac{E}{2\tau} = 155370 \times 10^6.$$

The *mechanical measure of the current intensity* is thus proportional

to *magnetic* as $1:155370 \times 10^6$,
to *electrodynamic* as $1:109860 \times 10^6$
$(= 1:155370 \times 10^6 \times \sqrt{1/2})$,
to *electrolytic* as $1:16573 \times 10^9$
$(= 1:155370 \times 10^6 \times 106\frac{2}{3})$.

3. Applications

Among the applications, which can be made by reducing the ordinary measure for current intensity to mechanical measure, the most important is the determination of the constants which appear in the fundamental electrical law, encompassing electrostatics, electrodynamics, and induction. According to this fundamental law, the effect of the amount of electricity e on the amount e' at distance r with relative velocity dr/dt and relative acceleration ddr/dt^2 equals

$$\frac{ee'}{rr}\left[1-\frac{1}{cc}\left(\frac{dr^2}{dt^2}-2r\frac{ddr}{dt^2}\right)\right],$$

and the constant c represents that relative velocity, which the electrical masses e and e' have and must retain, if they are not to act on each other any longer at all.

In the preceding section, the proportional relation of the magnetic measure to the mechanical measure was found to be

$$= 155370 \times 10^6 : 1;$$

in the second treatise on *electrical determination of measure*, the same proportion was found

$$= c\sqrt{2} : 4;$$

the equalization of these proportions results in

$$c = 439450 \times 10^6$$

units of length, namely, millimeters, thus a velocity of 59,320 miles per second.

The insertion of the values of c into the foregoing fundamental electrical law makes it possible to grasp, why the electrodynamic effect of electrical masses, namely

$$\frac{ee'}{rr}\frac{1}{cc}\left(\frac{dr^2}{dt^2}-2r\frac{ddr}{dt^2}\right)$$

compared with the electrostatic

$$\frac{ee'}{rr}$$

always seems infinitesimally small, so that in general the former only remains significant, when, as in galvanic currents, the electrostatic forces completely cancel each other in virtue of the neutralization of the positive and negative electricity.

Of the remaining applications, only the application to electrolysis will be briefly described here:

It was stated above, that in a current, which decomposes 1 milligram of water in 1 second,

$$106\frac{2}{3}\times 155370\times 10^6$$

positive units of electricity go in the direction of the positive current in that second through the cross-section of the current, and the same amount of negative electricity in the opposite direction.

The fact that in electrolysis, ponderable masses are moved, that this motion is elicited by electrical forces, which only react on electricity, not directly on the water, leads to the conception, that in the atom of water, the *hydrogen atom* possesses free positive electricity, the *oxygen atom* free negative electricity. Many reasons converge, why we do not want to think of an electrical motion in water without electrolysis, and why we assume that water is not in a state of allow electricity to flow through it in the manner of a conductor. Therefore, if we see in the one electrode just as much positive electricity coming from the water, as is delivered to the other electrode during the same time-interval by the current, then this positive electricity which manifests itself is that which belonged to the separated hydrogen particles.

If we take this standpoint, so that we thus link the entire electrical motion in electrolytes to the motion of the ponderable atoms, then it additionally emerges from the numbers obtained above, that the hydrogen atoms in 1 millimeter of water possess

$$106\frac{2}{3}\times 155370\times 10^{6}$$

units of free positive electricity, the oxygen atoms an equal amount of negative electricity.

From this it follows, secondly, that these amounts of electricity together signify the *minimum of neutral electricity*, which is contained in a milligram of water. Namely, if the atoms of water were still to possess neutral electricity beyond their free electricity, then the mass of neutral electricity in a milligram of water would be still greater.

Under the foregoing assumptions, we are also in a position to state the force with which the totality of the hydrogen particles of a mass of water is acted upon in the one direction, the totality of the oxygen particles in the opposite direction.

Imagine, for example, a cylindrical tube of 10/9 square millimeter cross-section, which is to serve as a decomposition cell, filled with a mixture of water and sulphuric acid of specific gravity 1.25, which thus contains in each 1-millimeter segment a milligram of water. Through Horsford, we know the proportional relation of the specific resistance of this mixture to that of silver, and through Lenz, the proportional relation of the resistance of silver to that of copper. In the treatises of the Koenigliche Gesellschaft der Wissenschaften in Goettingen (vol. 5, "Ueber die Anwendung der magnetischen Induction auf Messung der Inclination mit dem Magnetometer"), the resistance of copper is determined according to the absolute measure of the magnetic system. This makes it possible to additionally state, in *absolute magnetic measure*, the resistance which the water (under the influence of the admixed sulphuric acid) exerts in a 1-mm long segment of that cylindrical decomposition cell. This resistance, multiplied by the current intensity, the latter being expressed in magnetic measure, yields the electromotive force in relation to this small cell, likewise in the *magnetic* system of measure. However, the *magnetic* measure of the electromotive force is as many times smaller than the *mechanical*, as the magnetic measure of the current intensity is greater than the mechanical, and since this latter proportion is now known, that electromotive force calculated in magnetic measure can be transformed into mechanical measure simply by division by 155370×10^{6}. The number which results then signifies the *difference between the two forces*, of which in the direction of the current, the one acts to move *each single unit* of the free positive electricity in the hydrogen particles, the other to move *each single unit* of the free negative electricity in the oxygen particles, and therefore, in order to obtain the *entire force at work*, this number must still be multiplied by the total of units of the free positive or negative electricity, which is contained in the 1 millimeter-long wet cell, that is, in 1 milligram of water, namely, by

$$106\frac{2}{3}\times155370\times10^{6}.$$

If one carries out the calculation and presupposes that current intensity, at which 1 milligram of water is decomposed in 1 second, then one obtains a force difference

$$=2\times\left(106\frac{2}{3}\right)^{2}\times127476\times10^{6},$$

in which the unit of force is that force, which imparts to the unit of mass of 1 milligram a velocity of 1 millimeter in 1 second. Thus, if one divides by the intensity of gravity = 9,811, one obtains this force difference, expressed in weight

$$=2\times147830\times10^{6}\text{ milligrams}=2\times147830\text{ kg}=2\times2956\ \textit{Centner}$$

under the influence of gravity.

This result can be expressed in the following way: *If all hydrogen particles in 1 milligram of water were linked in a 1 millimeter-long string, and all oxygen particles in another string, then both strings would have to be stretched in opposite directions with the weight of 2,956 hundredweight, in order to produce a decomposition of the water at a rate such that 1 milligram of water would be decomposed in 1 second.*

One easily convinces oneself, that this stretching remains the same for a cell of 1 mm length but a different cross-section, but that it must be proportional to the length of the cell, and also proportional to the current intensity, that is, to the velocity of the electrolytic separation.

If, in the wet cell described above, we now see a pressure on the totality of hydrogen particles of the weight of 2,956 *centner*, and if no acceleration of motion occurs, which motion must, however, amount to 1,759 million miles per second, but rather the hydrogen continues with the constant velocity of ½ millimeter per second, then we are compelled to assume, that a force would be acting counter to the decomposition of the water, a force which increases with the velocity of the decomposition, so that in general, only that velocity of decomposition remains, at which the *force of resistance* is equal to the electromotive force, so that its effect on the totality of hydrogen particles in the milligram of water in the foregoing case likewise would equal the weight of 2,956 hundredweight. Namely, in that case, the ponderable particles would uniformly flow forth with the velocity attained.

It is natural, to seek the basis for this force of resistance in the *chemical forces of affinity*. Even though the concept of *chemical affinity* remains too indeterminate, for us to be able to derive from it, how the forces proceeding from this affinity increase with the *velocity of the separation*, nevertheless, it is interesting to see what colossal [*ungeheuren*] forces enter into operation, as are easily elicited by electrolysis.