Weber's Law and Mach's Principle

André K. T. Assis

1. Introduction

Recently we applied a Weber's force law for gravitation to implement quantitatively Mach's Principle (Assis 1989, 1992a). In this work we present a brief review of Weber's electrodynamics and analyze in greater detail the compliance of a Weber's force law for gravitation with Mach's Principle.

2. Weber's Electrodynamics

In this section we discuss Weber's original work as applied to electromagnetism. For detailed references of Weber's electrodynamics, see (Assis 1992b, 1994).

In order to unify electrostatics (Coulomb's force, Gauss's law) with electrodynamics (Ampère's force between current elements), W. Weber proposed in 1846 that the force exerted by an electrical charge q_2 on another q_1 should be given by (using vectorial notation and in the International System of Units):

$$\mathbf{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}}_{12}}{r_{12}^2}\left[1 - \frac{\dot{r}_{12}^2}{2c^2} + \frac{r_{12}\ddot{r}_{12}}{c^2}\right]. \qquad (1)$$

In this equation, $\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the permittivity of free space; the position vectors of q_1 and q_2 are $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively; the distance between the charges is

$$r_{12} \equiv |\mathbf{r}_1 - \mathbf{r}_2| = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2};$$

$\hat{\mathbf{r}}_{12} = (\mathbf{r}_1 - \mathbf{r}_2)/r_{12}$ is the unit vector pointing from q_2 to q_1; the radial velocity between the charges is given by $\dot{r}_{12} \equiv dr_{12}/dt = \hat{\mathbf{r}}_{12} \cdot \mathbf{v}_{12}$; and the

Einstein Studies, vol. 6: Mach's Principle: From Newton's Bucket to Quantum Gravity, pp. 159–171 © 1995 *Birkhäuser Boston, Inc. Printed in the United States.*

radial acceleration between the charges is

$$\ddot{r}_{12}=\frac{d\dot{r}_{12}}{dt}=\frac{d^2r_{12}}{dt^2}=\frac{[\mathbf{v}_{12}\cdot\mathbf{v}_{12}-(\hat{\mathbf{r}}_{12}\cdot\mathbf{v}_{12})^2+\mathbf{r}_{12}\cdot\mathbf{a}_{12}]}{r_{12}},$$

where

$$\mathbf{r}_{12}\equiv\mathbf{r}_1-\mathbf{r}_2,\ \mathbf{v}_{12}\equiv\frac{d\mathbf{r}_{12}}{dt},\ \mathbf{a}_{12}\equiv\frac{d\mathbf{v}_{12}}{dt}=\frac{d^2\mathbf{r}_{12}}{dt^2}.$$

Moreover, $c\equiv(\epsilon_0\,\mu_0)^{-1/2}$ is the ratio of electromagnetic and electrostatic units of charge ($\mu_0=4\pi\cdot 10^{-7}$ N/A^2 is the permeability of free space). This quantity c was first measured experimentally by W. Weber and Kohlrausch in 1856, when they found $c=3.1\cdot 10^8$ m/s. This was one of the first unambiguous and quantitative indications of an essential interconnection between electromagnetism and optics.

In 1848, Weber presented a potential energy U_{12} from which he could derive his force by $\mathbf{F}_{21}=-\hat{\mathbf{r}}_{12}dU_{12}/dr_{12}$:

$$U_{12}=\frac{q_1q_2}{4\pi\epsilon_0}\frac{1}{r_{12}}\left(1-\frac{\dot{r}_{12}^2}{2c^2}\right). \tag{2}$$

There is a Lagrangian L and a Hamiltonian H from which we can also derive his electrodynamics. For a system of two charges q_1 and q_2 of masses m_1 and m_2 interacting through Weber's force, we have a kinetic energy T_{12} and a Lagrangian energy S_{12} given by:

$$T_{12}=m_1\frac{\mathbf{v}_1\cdot\mathbf{v}_1}{2}+m_2\frac{\mathbf{v}_2\cdot\mathbf{v}_2}{2}=\frac{m_1v_1^2}{2}+\frac{m_2v_2^2}{2}, \tag{3}$$

$$S_{12}=\frac{q_1q_2}{4\pi\epsilon_0}\frac{1}{r_{12}}\left(1+\frac{\dot{r}_{12}^2}{2c^2}\right). \tag{4}$$

Note the change of sign in front of $\dot{r}_{12}^2$ in U_{12} and S_{12}.

Weber's Lagrangian and Hamiltonian are then given by

$$L\equiv T_{12}-S_{12}, \tag{5}$$

$$H\equiv\left(\sum_{k=1}^{6}\dot{q}_k\frac{\partial L}{\partial\dot{q}_k}\right)-L=T_{12}+U_{12}, \tag{6}$$

where $\dot{q}_k$, with k ranging from 1 to 6, represents the velocity components, namely, $\dot{x}_1$, $\dot{y}_1$, $\dot{z}_1$, $\dot{x}_2$, $\dot{y}_2$, and $\dot{z}_2$, respectively.

Weber's force can be obtained from S_{12} by the usual procedure. For instance, the x-component of $\mathbf{F}_{12}$ is given by

$$F_{21}^{x}=\frac{d}{dt}\frac{\partial S_{12}}{\partial \dot{x}_1}-\frac{\partial S_{12}}{\partial x_1}=\frac{q_1q_2}{4\pi\epsilon_0}\frac{x_1-x_2}{r_{12}^3}\left[1-\frac{\dot{r}_{12}^2}{2c^2}+\frac{r_{12}\ddot{r}_{12}}{c^2}\right]. \qquad (7)$$

The main properties of Weber's electrodynamics are:

A. It complies with Newton's action and reaction law, which means conservation of linear momentum for an isolated system of particles interacting through Weber's force and through other forces which also follow the law of action and reaction.

B. The force is always along the straight line connecting the two charges, which means conservation of angular momentum.

C. The force can be derived from the velocity dependent potential energy U_{12}, which means conservation of the total energy $E\equiv T_{12}+U_{12}$. Although Weber presented U_{12} in 1848, he proved the conservation of energy for his electrodynamics only in 1869 and 1871. In 1847, only one year after Weber had presented his force law (1), Helmholtz published his famous paper on the conservation of energy. In this work he showed that a force which depends on the distance and velocities of the interacting particles does not conserve energy, even if the force is a central one. This was the main objection that, from his first paper on electromagnetism of 1855/56, Maxwell advanced against Weber's electrodynamics and the reason that, in his own words, prevented him from considering Weber's theory as an ultimate one (Maxwell 1965a, 1965b). Maxwell was wrong, but he only changed his mind in 1871, after Weber's proof (Harman 1982). When he wrote the *Treatise* in 1873, he presented the new point of view that Weber's electrodynamics is consistent with the principle of conservation of energy (Maxwell 1954). Helmholtz's proof of 1847 does not apply to Weber's electrodynamics because Weber's force depends not only on the distance and velocity of the charges but also on their accelerations. This general case was not analyzed by Helmholtz at that time.

Other properties of Weber's law are:

D. When there is no relative motion between the interacting charges ($\dot{r}_{12}=0$ and $\ddot{r}_{12}=0$), we recover Coulomb's force and Gauss's law. So all electrostatics is embodied in Weber's electrodynamics.

E. Weber suceeded in deriving Faraday's law of induction (1831) from his force (Maxwell 1954).

F. Weber derived his force from Ampère's force (1823) exerted by the current element $I_2d\mathbf{l}_2$ on $I_1d\mathbf{l}_1$:

$$d^2\mathbf{F}_{21} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\hat{\mathbf{r}}_{12}\left[2(d\mathbf{l}_1 \cdot d\mathbf{l}_2) - 3(\hat{\mathbf{r}}_{12} \cdot d\mathbf{l}_1)(\hat{\mathbf{r}}_{12} \cdot d\mathbf{l}_2)\right]. \tag{8}$$

Alternatively we can postulate Weber's law and derive Ampère's force between current elements as a special case of Weber's electrodynamics. From Ampère's force (8) Maxwell derived what is known as Ampère's circuital law in 1856, twenty years after Ampère's death. Maxwell was the first to derive the circuital law even without the term with the displacement current.

The force between current elements usually found in the textbooks is due to Grassmann (1845) utilizing the Biot–Savart magnetic field $d\mathbf{B}_2$ of 1820, namely

$$d^2\mathbf{F}_{21} = I_1 d\mathbf{l}_1 \times d\mathbf{B}_2 = I_1 d\mathbf{l}_1 \times \left(\frac{\mu_1}{4\pi}\frac{I_2 d\mathbf{l}_2 \times \hat{\mathbf{r}}_{12}}{r_{12}^2}\right)$$

$$= -\frac{\mu_0}{4\pi}\frac{I_1 I_1}{r_{12}^2}\left[(d\mathbf{l}_1 \cdot d\mathbf{l}_2)\hat{\mathbf{r}}_{12} - (d\mathbf{l}_1 \cdot \hat{\mathbf{r}}_{12})d\mathbf{l}_2\right]. \tag{9}$$

Ampère's force (8) complies with the action and reaction law in the strong form for any independent orientation of each current element, while this is not valid in general for Grassmann's force (9). Both expressions give the same result for the force of a closed current loop of arbitrary form on a current element of another circuit. In the last ten years many experiments have been performed trying to distinguish (8) and (9) in situations involving a single circuit (for instance, measuring and calculating the force and tension on a mobile part of a closed circuit due to the remainder of the circuit). Although most experiments seem to favor Ampère's force over Grassmann's one, the situation is not yet completely clear, and more experiments and theoretical analysis are desirable before a final conclusion can be drawn. For references on this topic see (Assis 1989, 1992b, 1994).

It should be remembered that Maxwell knew both expressions, (8) and (9). When comparing these assumptions he said that "Ampère's is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them" (Maxwell 1954, vol. 2, § 527, p. 174).

The last property of Weber's law to be discussed here is undoubtedly one of the most important of them. It is also closely related to Mach's Principle:

G. The law depends only on the relative distance between the particles, r_{12}, on the relative velocity between them, $\dot{r}_{12} = dr_{12}/dt$, and on the relative radial acceleration between them,

$$\ddot{r}_{12} = d\dot{r}_{12}/dt = d^2 r_{12}/dt^2.$$

This is what we call a relational theory. These terms have the same value in all frames of reference, even for noninertial ones.

This is a distinguishing feature of Weber's electrodynamics. In the other formulations of electromagnetism the terms in the velocity and acceleration of the particles which are relevant depend on the velocities or accelerations of the charges either relative to a material medium like the ether, or relative to an inertial frame of reference. This last situation is typical of Lorentz's force law, $\mathbf{F}=q\mathbf{E}+q\mathbf{v}\times\mathbf{B}$, where $\mathbf{v}$ is the velocity of the charge q relative to an arbitrary inertial frame of reference (and not, for instance, relative to the laboratory or to the magnet which generated the magnetic field $\mathbf{B}$).

After this short review we shall discuss the relation of Weber's electrodynamics to Mach's Principle.

3. The Mach–Weber Model

In order to implement quantitatively Mach's Principle we need to modify Newton's law of gravitation by including terms which depend on the velocity and acceleration between the interacting bodies. This was never done by Mach himself. In our opinion the best model in this direction seems to be some kind of Weber's law for gravitation. In the first place this would comply with Mach's idea that only relative positions and motions are important, as this force depends only on r_{12}, $\dot{r}_{12}$, and $\ddot{r}_{12}$. It also depends on the accelerations of the source and test bodies. So it has embodied in it the possibility of deriving $m\mathbf{a}$, the centrifugal and Coriolis forces as real gravitational forces arising from the relative acceleration of the test body and the remainder of the universe.

Here we list some (but not all) people who have worked with this model. The first to propose a Weber's law for gravitation seems to have been G. Holzmüller in 1870 (North 1965, p. 46). Then Tisserand, in 1872, studied a Weber's law for gravitation and its application to the precession of the perihelion of the planets (Tisserand 1872, 1895). Weber himself and Zölner obtained this law as applied to gravitation around 1876, when implementing the idea of Young and Mossotti of deriving gravitation from electromagnetism (Assis 1992b; Woodruff 1976). Later on Paul Gerber obtained essentially the same potential energy up to second order in $1/c$ (Gerber 1898). He obtained this law independently, following ideas of retarded time, without discussing

Weber's work. He also studied the precession of the perihelion of the planets. Gerber's work was criticized by Seeliger (Seeliger 1917), who was aware of Weber's electrodynamics. The work of Tisserand applying a Weber's law for gravitation in celestial mechanics was also discussed by Poincaré in a course which he delivered at the Faculté des Sciences de Paris during 1906–1907 (Poincaré 1953, see especially p. 125 and Chap. IX, pp. 201–203, "Loi de Weber"). None of the authors tried to implement Mach's Principle with these force laws.

Although Mach dealt with many branches of physics (mechanics and gravitation, optics, thermodynamics), we are not aware that he ever mentioned Weber's electrodynamics. We also do not know any reference of Einstein to Weber's force or potential energy. The first to suggest a Weber's law for gravitation in order to implement Mach's Principle seems to have been I. Friedlaender in 1896 (Friedlaender and Friedlaender 1896, p. 17, footnote, p. 310 in this volume). They seem to have been also the first to suggest that inertia should be related to *gravitation*. Höfler in 1900, although opposing Mach, mentioned Weber's electrodynamics when discussing Mach's Principle (Norton 1995). Hofmann in 1904 suggested a kinetic energy that depended on the product of the masses, on a function of the distance between the interacting masses, and on the square of their relative speed, which is somewhat similar to Weber's potential energy when applied to gravitation (this volume, p. 128). In this century we have Reissner and Schrödinger considering relational quantities in gravitation to implement Mach's Principle (Reissner 1914, 1915, this volume, p. 134; Schrödinger 1925, this volume, p. 147). They arrived independently at a potential energy very similar to that of Weber, apparently without being aware of Weber's electrodynamics. In 1933, we have Przeborski discussing Weber's law and other expressions in connection with Newton's second law of motion, although not analyzing Mach's Principle directly (Przeborski 1933). More recently we have Sciama (1953). Although he made an analogy between gravitation and electromagnetism, he did not work with a relational force law, and his expression did not even comply with Newton's action and reaction principle. He also did not mention Weber's electrodynamics. Brown was closer to this idea, although his force law is different from Weber's one (Brown 1955, 1982). Moon and Spencer published an important work on this topic (Moon and Spencer 1959), although they did not consider Weber's law or relational quantities. Edwards worked explicitly with relational quantities and with analogies between electromagnetism and gravitation (Edwards 1974). Once more Weber's electrodynamics is not mentioned. Barbour and Bertotti opened new lines of research working not only with relational quantities but with intrinsic derivatives and with the relative configuration space (RCS) of the universe (Barbour 1974; Barbour and Bertotti 1977, 1982). Eby worked along this line and studied the precession of the perihelion of the planets (Eby 1977). Although he worked essentially with a Weber's Lagrangian, he did not mention Weber's work. Treder, von Borzeszkowski, van der Merwe, Yourgrau, and collaborators have worked with and discussed explicitly a Weber's force applied to gravitation. References to their original works and to other authors can be found in (Treder 1975; Treder, von Borzeszkowski, van der Merwe, and Yourgrau 1980). Ghosh worked with closely related ideas, although he was not aware of Weber's force (Ghosh 1984, 1986, 1991). More recently we have Wesley and a direct use of Weber's law (Wesley 1990). He also worked with a potential similar to Schrödinger's potential energy (Schrödinger 1925), without being aware of that work.

Although we could quote many other authors and papers, we stop here. This short list gives an idea of the continuing effort and research that has been performed by many important people along this line (trying to implement quantitatively Mach's Principle by some kind of Weber's law). We are following these ideas, although we were not aware of many of these works when we began. Here we present how we deal with this subject (Assis 1989, 1992a).

Our basic idea is to begin with a gravitational potential energy between two particles given by

$$U_{12} = -H_g \frac{m_{g1} m_{g2}}{r_{12}} \left(1 - \frac{\xi}{2} \frac{\dot{r}_{12}^2}{c^2} \right) \exp(-\alpha r_{12}). \tag{10}$$

In this expression, H_g is an arbitrary constant, $m_{g1,2}$ are gravitational masses, ξ is a dimensionless constant, and α gives the characteristic length of the gravitational interaction. Newton's potential energy is (10) with $H_g = G$, $\xi = 0$, and $\alpha = 0$.

The first to propose an exponential decay in the gravitational potential energy were Seeliger and Neumann, in 1895–1896. What they proposed would be equivalent to (10) with $H_g = G$ and $\xi = 0$. An exponential term in Newton's gravitational force (but not in the potential) had been proposed much earlier by Laplace, in 1825. For references and further discussion see (Assis 1992a; North 1965, pp. 16–18; Laplace 1969; Seeliger 1895). In this century there is a remarkable paper by W. Nernst proposing an exponential decay in gravitation (Nernst 1937). These exponential decays have been proposed as an absorption of gravity due to the intervening medium, in analogy with the propagation of light. In this case α would depend on the amount and distribution of the intervening matter in the straight line between m_{g1} and m_{g2}. Alternatively it has also been proposed to solve some gravitational paradoxes arising in an infinite and homogeneous universe (indefinite value of the potential or of the gravitational force). In this last situation α may be considered as a universal constant irrespective of the medium between m_{g1} and m_{g2}.

To our knowledge we were the first to propose the exponential decay in a Weberian potential (Assis 1992a).

To simplify the analysis in this work, we will consider the arbitrary constant H_g as equal to Newton's gravitational constant G. Moreover we will treat α as a constant irrespective of the medium between the particles 1 and 2. Its value will be taken as $\alpha = H_0/c$, where H_0 is Hubble's constant (Assis 1992a). We will also take $\xi = 6$, as in our previous work (Assis 1989, 1992a).

The force exerted by m_{g2} on m_{g1} can be obtained utilizing $\mathbf{F}_{21} = -\hat{\mathbf{r}}_{12} dU_{12}/dr_{12}$. This yields:

$$\mathbf{F}_{21} = -G\frac{m_{g1}m_{g2}}{r_{12}^2}\hat{\mathbf{r}}_{12}\left[1 - \frac{\xi}{2}\frac{\dot{r}_{12}^2}{c^2} + \xi\frac{r_{12}\ddot{r}_{12}}{c^2} + \frac{H_0}{c}r_{12}\left(1 - \frac{\xi}{2}\frac{\dot{r}_{12}^2}{c^2}\right)\right]\exp(-H_0 r_{12}/c). \tag{11}$$

We now integrate this expression for a particle of gravitational mass m_{g1} interacting with an isotropic, homogeneous and infinite universe. Its average gravitational matter density is represented by ρ_0. In order to integrate we utilize spherical coordinates and replace m_{g2} by $\rho_0 r_2^2 \sin\theta_2 \, dr_2 d\theta_2 d\varphi_2$. We integrate from $\varphi_2 = 0$ to 2π, from $\theta_2 = 0$ to π, and from $r_2 = 0$ to infinity. The procedure is the same as in (Assis 1989, 1992a). We perform the integration in a frame of reference relative to which the universe as a whole (the set of distant galaxies) has an overall translational acceleration $\mathbf{a}_u$ and is rotating with an angular velocity $\boldsymbol{\omega}_u(t)$. Relative to this arbitrary frame of reference, the particle m_{g1} is located at the position $\mathbf{r}_1$ and has a velocity $\mathbf{v}_1 = d\mathbf{r}_1/dt$ and acceleration $\mathbf{a}_1 = d^2\mathbf{r}_1/dt^2$. The final result of the integration is found to be

$$\mathbf{F}_{u1} = -Am_{g1}\left[\mathbf{a}_1 + \boldsymbol{\omega}_u \times (\boldsymbol{\omega}_u \times \mathbf{r}_1) - 2\boldsymbol{\omega}_u \times \mathbf{v}_1 - \frac{d\boldsymbol{\omega}_u}{dt} \times \mathbf{r}_1 - \mathbf{a}_u\right]. \tag{12}$$

In this expression

$$A = \frac{4\pi}{3} H_g \frac{\xi}{c^2} \rho_0 \int_0^\infty r_2 \exp(-\alpha r_2) dr_2 = \frac{4\pi}{3} G\xi \frac{\rho_0}{H_0^2}. \tag{13}$$

In Newtonian mechanics, this expression is zero.

To complete the formulation of a Machian dynamics, we need the principle of dynamical equilibrium (Assis 1989). According to this principle, the sum of all forces of any nature (gravitational, electromagnetic, elastic, nuclear, etc.) on any particle is always zero in all coordinate frames, even when the particle is in motion and accelerated. We represent by $\Sigma_{j=1}^{N} \mathbf{F}_{j1}$ the resultant force acting on m_{g1} due to N local bodies j (like the gravitational force of the earth and the sun, contact forces, electromagnetic forces, friction forces, etc.). The principle of dynamical equilibrium can then be expressed as:

$$\sum_{j=1}^{N} \mathbf{F}_{j1} + \mathbf{F}_{u1} = 0. \tag{14}$$

Utilizing (12) this can be written as

$$\frac{\sum_{j=1}^{N} \mathbf{F}_{j1}}{A} - m_{g1} \boldsymbol{\omega}_u \times (\boldsymbol{\omega}_u \times \mathbf{r}_1) + 2m_{g1} \boldsymbol{\omega}_u \times \mathbf{v}_1$$

$$+ m_{g1} \frac{d\boldsymbol{\omega}_u}{dt} \times \mathbf{r}_1 + m_{g1} \mathbf{a}_u = m_{g1} \mathbf{a}_1. \tag{15}$$

This is essentially Newton's second law of motion with 'fictitious' forces. In the Mach–Weber model these are real gravitational forces which arise in any frame of reference in which the universe as a whole has a translational acceleration $\mathbf{a}_u$ and is rotating as a whole with an angular velocity $\boldsymbol{\omega}_u$. The proportionality between Newton's inertial and gravitational masses (the principle of equivalence) is derived at once in this model as the right-hand side of (15) arose from the gravitational interaction (12) of m_{g1} with the isotropic matter distribution surrounding it. The constant A must be exactly equal to 1, and this is known to be approximately true since the 1930s with Dirac (Assis 1989, 1992a). Equation (15) takes its simplest form in a frame of reference in which the universe at large is essentially stationary ($\mathbf{a}_u = 0$, $\boldsymbol{\omega}_u = 0$, $d\boldsymbol{\omega}_u/dt = 0$). This explains the coincidence (in Newtonian mechanics) that the frame of the fixed stars is the best inertial frame we have, namely, a frame in which there are no fictitious forces (Schiff 1964).

ACKNOWLEDGMENTS

The author wishes to thank Dr. J. B. Barbour for calling his attention to the papers by Reissner and Schrödinger. He is also grateful to the Organizing Committee of the International Conference on Mach's Principle for financial support.

REFERENCES

Assis, André K. T. (1989). "On Mach's Principle." *Foundations of Physics Letters* 2: 301–318.

Assis, André K. T. (1992a). "On the Absorption of Gravity." *Apeiron* 13: 3–11.

Assis, André K. T. (1992b). "Deriving Gravitation from Electromagnetism." *Canadian Journal of Physics* 70: 330–340.

Assis, André K. T. (1994). *Weber's Electrodynamics*. Dordrecht: Kluwer.

Barbour, Julian B. (1974). "Relative-Distance Machian Theories." *Nature* 249: 328–329. Misprints corrected in *Nature* 250: 606 (1974).

Barbour, Julian B. and Bertotti, Bruno (1977). "Gravity and Inertia in a Machian Framework." *Nuovo Cimento B* 38: 1–27.

Barbour, Julian B. and Bertotti, Bruno (1982). "Mach's Principle and the Structure of Dynamical Theories." *Proceedings of the Royal Society of London A* 382: 295–306.

Brown, G. B. (1955). "A Theory of Action-at-a-Distance." *Proceedings of the Physical Society B* 68: 672–678.

Brown, G. B. (1982). *Retarded Action-at-a-Distance*. Luton: Cortney Publications, Chapter 7, "The Law of Gravitation and the Origin of Inertia."

Eby, Peter B. (1977). "On the Perihelion Precession as a Machian Effect." *Lettere al Nuovo Cimento* 18: 93–96.

Edwards, W. F. (1974). "Inertia and an Alternative Approach to the Theory of Interactions." *Proceedings of the Utah Academy of Sciences, Arts, and Letters* 51 (2): 1–7.

Friedlaender, Benedict and Friedlaender, Immanuel (1896). *Absolute oder relative Bewegung?* Berlin: Leonhard Simion. [Partial transl. in this volume p. 114 and p. 309.]

Gerber, Paul (1898). "Die räumliche und zeitliche Ausbreitung der Gravitation." *Zeitschrift fur Mathematik und Physik II* 43: 93–104.

Ghosh, Amithaba (1984). "Velocity Dependent Inertial Induction: An Extension of Mach's Principle." *Pramana Journal of Physics* 23: L671–L674.

Ghosh, Amithaba (1986). "Velocity-Dependent Inertial Induction and Secular Retardation of the Earth's Rotation." *Pramana Journal of Physics* 26: 1–8.

Ghosh, Amithaba (1991). "Velocity-Dependent Inertial Induction: A Possible Tired-Light Mechanism." *Apeiron* 9–10: 35–44.

Harman, P. M. (1982). *Energy, Force, and Matter – The Conceptual Devel-*

opment of Nineteenth-Century Physics. Cambridge: Cambridge University Press, pp. 96-97.

Laplace, Pierre S. (1969). *Celestial Mechanics*, Vol. 5. New York: Chelsea Publishing Company; reprint of Vol. V of the *Traité de Mécanique Céleste*, first published in 1825 and reprinted in 1882 as Volume 5 of Laplace's collected works, Book XVI, Chapter IV, pp. 448–452.

Mach, Ernst (1960). *The Science of Mechanics: A Critical and Historical Account of Its Development*. LaSalle, Illinois: Open Court.

Maxwell, James C. (1954). *A Treatise on Electricity and Magnetism*. New York: Dover, Volume 2, Chapter 23.

Maxwell, James C. (1965a). "On Faraday's Lines of Force." In *The Scientific Papers of James Clerk Maxwell*. W. D. Niven, ed. New York: Dover, pp. 155–229, see especially p. 208.

Maxwell, James C. (1965b). "A Dynamical Theory of the Electromagnetic field." In *The Scientific Papers of James Clerk Maxwell*, W. D. Niven, ed. New York: Dover, pp. 526–597, see especially p. 527.

Moon, Parry and Spencer, Domina E. (1959). "Mach's Principle." *Philosophy of Science* 26: 125–134.

Nernst, Wilhelm (1937). "Weitere Prüfung der Annahme eines stationären Zustandes im Weltall." *Zeitschrift fur Physik* 106: 633–661.

Newton, Isaac (1952). *Mathematical Principles of Natural Philosophy*. In *Great Books of the Western World*, Vol. 34. Chicago: Encyclopaedia Britannica, translated from Latin by Andrew Motte and revised by Florian Cajori.

North, John D. (1965). *The Measure of the Universe – A History of Modern Cosmology*. Oxford: Clarendon Press.

Norton, John D. (1995). "Mach's Principle before Einstein." This volume, p. 9.

Poincaré, Henri (1953). "Les Limites de la Loi de Newton." *Bulletin Astronomique* 17: 121–269.

Przeborski, M. A. (1933). "Sur les Forces Dépendent des Accélérations." *Comptes Rendues de l'Academie des Sciences de Paris* 197: 300–302.

Reissner, Hans (1914). "Über die Relativität der Beschleunigungen in der Mechanik." *Physikalishe Zeitschrift* 15: 371–375. [Transl. in this volume, p. 134.]

Reissner, Hans (1915). "Über eine Möglichkeit die Gravitation als unmittelbare Folge der Relativität der Trägheit abzuleiten." *Physikalishe Zeitschrift* 16: 179–185. [Extracts translated in this volume, p. 134.]

Schiff, Leonard I. (1964). "Observational Basis of Mach's Principle." *Reviews of Modern Physics* 36: 510–511.

Schrödinger, Erwin (1925). "Die Erfüllbarkeit der Relativitatsforderung in der klassischen Mechanik." *Annalen der Physik* 77: 325–336.

Sciama, Dennis W. (1953). "On the Origin of Inertia." *Monthly Notices of the Royal Astronomical Society* 113: 34–42.

Seeliger, Hugo (1895). "Ueber das Newton'sche Gravitationsgesetz." *Astronomische Nachrichten* 137: 129–136.

Seeliger, Hugo (1917). "Bemerkung zu P. Gerbers Aufsatz: 'Die Fortpfanzungsgeschwindigkeit der Gravtitation'." *Annalen der Physik* 53: 31–32.

Tisserand, François Félix (1872). "Sur le Mouvement des Planètes autour du Soleil, d'après la Loi Électrodynamique de Weber." *Comptes Rendues de l'Academie des Sciences de Paris* 75: 760–763.

Tisserand, François Félix (1895). *Traité de Mécanique Céleste*. Paris: Gauthier-Villars, Vol. 4, Chap. 28 (Vitesse de propagation de l'attraction), pp. 499–503, "Loi d'Attraction Conforme à la Loi Électrodynamique de Weber."

Treder, Hans J. (1975). "The Post-Newtonian Effect of Gravitation and the Retardation of the Gravitational Potential in Classical and Relativistic Theories of Gravitation." *Annalen der Physik* 32: 338–350.

Treder, Hans J., von Borzeszkowski, H. H., van der Merwe, Alwyn and Yourgrau, W. (1980). *Fundamental Principles of General Relativity Theories*. New York: Plenum, Chapter 2.

Wesley, James P. (1990). "Weber Electrodynamics: Part III. Mechanics, Gravitation." *Foundations of Physics Letters* 3: 581–605.

Woodruff, A. E. (1976). "Weber, Wilhelm Eduard." In *Dictionary of Scientific Biography*. C. C. Gillispie, ed. New York: Scribner, Vol. 14, pp. 203–209.

Discussion

Vucetich: When you introduce the expression for the force, you explicitly introduce Hubble's constant, which is not a constant, generally, but varies in time. So do you get a varying gravitational constant?

Assis: The Hubble constant in this model is introduced as the term in the exponential decay. So if you write that expression in terms of Hubble's constant, you have two choices: If the universe is expanding and so on, you get that the Newtonian gravitational constant is related to Hubble's constant. If one is varying, the other is also varying. But if the universe has no expansion, and Hubble's law of red shift has another origin, like tired light or any other thing, then Hubble's constant and Newton's gravitational constant will be constant in time. So that depends on the origin of the red shift.

Lynden-Bell: You take a totally isotropic universe.

Assis: No, I assume you can always divide the universe into two parts – one anisotropic, and one isotropic.

Lynden-Bell: Yes, but I think if you take a small but significant thing like the center of the Galaxy, or the Great Attractor, or something like that, which is far away, and in the system, you'll find that the mass is slightly anisotropic.

Assis: Not necessarily, because this anisotropy may also appear in the other constants, which you apply in the force. So, like Dicke (1961, 1964) said, the effects may cancel out in the end.

Lynden-Bell: Well, that might happen, but I think in a purely gravitational situation, I don't think it does [see, for example, (Nordtvedt 1975)].

Brill: If you try to implement into your scheme the principle of relativity, according to which influences take a finite time to propagate, would you then need to introduce advanced potentials? If I start accelerating now, then I see the distant universe accelerate now; but because what I see now happened earlier, the universe must have started accelerating a long time ago.

Assis: Yes, what I would say is that only recently have people begun to introduce retardation in Weber's law. There is a paper by Wesley (1990), who introduced that since 1987. Not only in electrodynamics, but also in gravitation. And so the situation is still open with regard to what we will get with retardation in Weber's law applied to gravitation and electrodynamics. But this is a new area of research which is being performed nowadays, so I can't answer it now.

REFERENCES

Dicke, Robert H. (1961). "Experimental Tests of Mach's Principle." *Physical Review Letters* 7: 359–360. Reprinted in (Dicke 1964).

Dicke, Robert H. (1964). *The Theoretical Significance of Experimental Relativity*. London: Blackie and Son Ltd.

Nordtvedt, Kenneth (1975). "Anisotropic Gravity and the Binary Pulsar PSR 1913+16." *Astrophysical Journal* 202: 248–249.

Wesley, James P. (1990). "Weber Electrodynamics." *Foundations of Physics Letters* 3: 443–490.