In: Advanced Electromagnetism - Foundations, Theory and Applications, T. W. Barrett and D. Grimes (eds.), World Scientific, Singapore, 1995, pp. 314-331.

Gravitation as a Fourth Order Electromagnetic Effect

A. K. T. Assis*†

Instituto de Física 'Gleb Wataghin'
Universidade Estadual de Campinas - Unicamp
13083-970 Campinas, São Paulo, Brasil

Abstract

We present a generalised Weber's law for electromagnetism including terms of fourth and higher orders in $1/c$. These extra terms when applied to the force between two neutral dipoles yield an equivalent to Newton's law of universal gravitation as a fourth order electromagnetic effect.

Key Words: Weber's electrodynamics, Weber's law, Weber force, Newton's law of gravitation, unified theories.

PACS: 04.50.+h Unified field theories and other theories of gravitation
12.10.Gq Unified theories and models including gravitation
12.25.+e Models for gravitational interactions
41.90.+e Other topics in electricity and magnetism

*Internet: ASSIS@IFI.UNICAMP.BR

†Also Collaborating Professor at the Department of Applied Mathematics, IMECC, State University of Campinas, 13081-970 Campinas, SP, Brazil.

1 Introduction

One of the main goals of physicists in this century is to unify gravitation and electromagnetism. The subjects of electrostatics (interaction of electrical charges at rest), magnetostatics (interaction of magnets with one another and with the Earth) and galvanism (study of electrical currents) were unified in a single framework during the last century. Despite many efforts, gravitation has always remained separated from the other fundamental interactions of nature. But we believe it is possible to unify these two branches of knowledge.

There are many indications that gravitation should be related to electromagnetism. The first one is that Newton's law of gravitation and Coulomb's force are very similar. Both expressions are proportional to a product of an intrinsic property of the bodies (their gravitational masses or their electrical charges), fall as $1/r^2$, and are along the line connecting the bodies. There are two main differences, however. The first one is that up to now we only know one kind of gravitational mass, while two kinds of charge are known: positive and negative. Föppl suggested in 1897 that we might have as well two kinds of mass, positive and negative: See [1], Chapter II, Section III, p. 234. Two positive masses would attract one another, as would two negative masses. On the other hand a positive and a negative mass would repel one another. This is an ingenious idea, although up to now it has not received experimental confirmation. The second difference between Coulomb's force and Newton's force is in the order of magnitude. Two fundamental particles known to us are the electron and the proton. But the gravitational force between them is 10^{40} times smaller than their electrical attraction at the same distance. This suggests that gravitation could be derived from electromagnetism as a residual effect. In this work we present an explanation for this remarkable fact.

Another indication of a connection between gravitation and electromagnetism is that all neutral bodies known to us like an atom or a neutron have been broken in smaller charged particles like protons and electrons. This indicates that all bodies may be composed of charged elements. In this work we explore this property showing that an attractive force like gravitation arises as a residual electromagnetic force between neutral electrical dipoles.

A third indication of this connection comes from the conservation and transformation of energy. In modern hydroelectric power stations we transform gravitational potential energy into electromagnetic energy. The opposite effect happens in any electromagnetic device designed to raise weights like any elevator powered by electricity. This proves that these two interactions may be converted into one another.

What remains to be shown is a theoretical derivation of gravitation from electromagnetism. This is the subject of this work.

The main concept we try to explore here is the development of gravitation as a fourth order electromagnetic effect. The basic electromagnetic model with which we work is Weber's law. For this reason we begin by presenting this theory, [2].

2 Weber's Electrodynamics

Wilhelm Weber (1804–1891) presented his force law in 1846. At this time Coulomb's force of 1785, Ampère's force between current elements which arose from his experiments in the period 1820-1827 and Faraday's law of induction (1831) were known. Coulomb's force can be written in the International System of Units MKSA and in vectorial notation as:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}}{r^2} \ . \tag{1}$$

In this equation $\vec{F}$ is the force exerted by charge q_2 on charge q_1. They are located at $\vec{r}_2$ and $\vec{r}_1$, respectively. Moreover, $\varepsilon_o = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$, $r = |\vec{r}_1 - \vec{r}_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$ is the distance between the charges and $\hat{r} = (\vec{r}_1 - \vec{r}_2)/r$ is the unit vector pointing from q_2 to q_1. This force is similar to Newton's law of universal gravitation (1687) as it complies with the law of action and reaction, is along the straight line connecting the bodies and falls as $1/r^2$. Newton's force of gravitation is given by:

$$\vec{F} = -Gm_1m_2\frac{\hat{r}}{r^2} \ . \tag{2}$$

Here $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$, while m_1 and m_2 are the gravitational masses of bodies 1 and 2, respectively.

These two force laws can be derived from potential energies given by, respectively:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{1}{r} \ , \tag{3}$$

$$U = -G\frac{m_1 m_2}{r} \ . \tag{4}$$

The standard procedure to derive the force from the potential is through

$$\vec{F} = -\hat{r}\frac{dU}{dr} \ . \tag{5}$$

It is usually believed that Coulomb derived his force from the result of his experiments with the torsion balance. But in an interesting article published by Heering it has been shown that Coulomb was led to this expression more in analogy with Newton's law of gravitation than as a result of his few measurements: [3].

Ampère's force (1826) exerted by the current element $I_2 d\vec{l}_2$ on $I_1 d\vec{l}_1$ can be written as:

$$d^2\vec{F} = -\frac{\mu_o}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} \left[2(d\vec{l_1} \cdot d\vec{l_2}) - 3(\hat{r} \cdot d\vec{l_1})(\hat{r} \cdot d\vec{l_2}) \right] \ . \tag{6}$$

In this equation $\mu_o = 4\pi \times 10^{-7}\ kgmC^{-2}$. This force also complies with the law of action and reaction: it is along the straight line connecting the current elements and falls as $1/r^2$.

Nowadays the only expression which appears in the textbooks for the force between current elements is Grassmann's one (1845), which is based on Biot-Savart's magnetic field $d\vec{B}_2$ of 1820, namely:

$$d^2\vec{F} = I_1 d\vec{l_1} \times d\vec{B}_2 = I_1 d\vec{l_1} \times \left(\frac{\mu_o}{4\pi} \frac{I_2 d\vec{l_2} \times \hat{r}}{r^2} \right) \ . \tag{7}$$

Operating the double vectorial product yields:

$$d^2\vec{F} = -\frac{\mu_o}{4\pi} \frac{I_1 I_2}{r^2} \left[(d\vec{l_1} \cdot d\vec{l_2})\hat{r} - (d\vec{l_1} \cdot \hat{r}) d\vec{l_2} \right] \ . \tag{8}$$

In the last ten years there has been a great controversy surrounding these two force laws. Are they always equivalent? Is the exploding wire phenomenon and similar experiments due to Ampère's tension? As there are two contributions in this book dealing exclusively with this subject, see the works by Peter Graneau and Rémi Saumont, we will not consider it here anymore. For interested readers we suggest two excellent books dealing with this subject: [4] and [5].

Weber's idea in 1845 and 1846 was to unify electrostatics with electrodynamics, namely, to derive Ampère's force, (6), from a generalization of Coulomb's force, (1). If we utilize that $\mu_o = 1/\varepsilon_o c^2$ and the fact that conduction currents are charges in motion ($Id\vec{l} \to dq\vec{v}$), we can see that Ampère's force is similar to Coulomb's force multiplied by terms of the order $v_1 v_2/c^2$. So in order to derive Ampère's force Weber needed to generalize Coulomb's law including terms of the second order in $1/c$. The result he obtained is the following:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_o} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\ddot{r}}{c^2} \right) \ . \tag{9}$$

In this equation

$$\dot{r} = \frac{dr}{dt} = \hat{r}_{12} \cdot \vec{v}_{12} \ , \tag{10}$$

$$\ddot{r} = \frac{d^2 r}{dt^2} = \frac{d\dot{r}}{dt} = \frac{\vec{v}_{12} \cdot \vec{v}_{12} - (\hat{r}_{12} \cdot \vec{v}_{12})^2 + \vec{r}_{12} \cdot \vec{a}_{12}}{r_{12}} \ , \tag{11}$$

where

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 \ . \tag{12}$$

$$\vec{v}_{12} = \frac{d\vec{r}_{12}}{dt} = \frac{d}{dt}(\vec{r}_1 - \vec{r}_2) \ , \tag{13}$$

$$\vec{a}_{12} = \frac{d^2\vec{r}_{12}}{dt^2} = \frac{d\vec{v}_{12}}{dt} \ . \tag{14}$$

The work where he presented his force for the first time was published in 1846. A shorter version of this paper has already been translated to English: [6]. There are other important papers of Weber related to this subject which have also been translated to English: [7] and [8]. His complete works were published between 1892 and 1894, in 6 volumes: [9], [10], [11], [12], [13] and [14]. A complete discussion of Weber's theory applied to electromagnetism and gravitation can be found in the recent book *Weber's Electrodynamics*, [2].

The electromagnetic constant c (which in the MKSA system is written as $1/\sqrt{\mu_o \varepsilon_o}$) appeared for the first time in physics in this force given by Weber in 1846. It is the ratio of electromagnetic to electrostatic units of charge. He was also the first to measure this quantity in a joint work with Kohlrausch in 1856 when they found its value to be $3.1 \times 10^8 \ ms^{-1}$. A good description of this experimental work can be found in [15].

In 1856-1857 Weber and G. Kirchhoff, working independently, obtained the wave equation describing the propagation of electromagnetic signals along wires. This was previous to the work of J. C. Maxwell in 1860-1864. And the amazing fact is that they determined correctly that for wires of negligible resistance the signal (fluctuations in the surface charge density of the wire, or fluctuations in the current or potential along the wire) would travel at light velocity! This was obtained with the action at a distance theory of Weber without time retardation, without the displacement current and without the concept of a medium or ether surrounding the wire. A discussion of their work can be found in: [16], [17] and [18], Vol. 1, pp. 144–146 and 296–297. Kirchhoff's three papers dealing with this subject (one of 1850 and two of 1857) have already been translated to English: [19], [20] and [21].

In his work of 1846 Weber also succeeded in deriving Faraday's law of induction from his force law. Two years later he presented a velocity dependent potential energy U from which his force could be derived from the standard procedure:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{1}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right) \ . \tag{15}$$

This was the first example in physics of a potential energy which depended on the velocity of the interacting particles. To derive Eq. (9) from (15) we apply (5) and utilize that

$$\frac{d\dot{r}^2}{dr} = 2\dot{r}\frac{d\dot{r}}{dr} = 2\dot{r}\frac{d\dot{r}}{dt}\frac{dt}{dr} = 2\ddot{r} \ . \tag{16}$$

From 1869 to 1871 Weber proved that his force law complied with the principle of conservation of energy. This overcame a great difficulty in the acceptance of Weber's theory, especially with the followers of Helmholtz, as he never accepted Weber's law.

Let us discuss this briefly: Weber presented his force in 1846. The next year Helmholtz presented his influential work on the conservation of energy. This paper has already been translated to English: [22]. The main results of his paper were stated as follows (our words are between square brackets):

> The preceding propositions may be collected as follows:
>
> 1. Whenever natural bodies act upon each other by attractive or repulsive forces, which are independent of time and velocity, the sum of their *vires vivae* [kinetic energies, or $mv^2/2$] and tensions [potential energies] must be constant; the maximum quantity of work which can be obtained is therefore a limited quantity.
>
> 2. If, on the contrary, natural bodies are possessed of forces which depend upon time and velocity, or which act in other directions than the lines which unite each two separate material points, for example, rotatory forces, then combination of such bodies would be possible in which force might be either lost or gained *ad infinitum*.

This was understood by Maxwell and many others as implying that Weber's electrodynamics did not comply with the principle of conservation of energy. For instance, in his first paper on electromagnetism, [23], Maxwell discussed Weber's electrodynamics, after presenting Faraday's ideas of an electrotonic state with a mathematical foundation. He said (our emphasis in boldface):

> There exists however a professedly physical theory of electro-dynamics, which is so elegant, so mathematical, and so entirely different from anything in this paper, that I must state its axioms, at the risk of repeating what ought to be well known. It is contained in M. W. Weber's *Electro-dynamic Measurements*, and may be found in the Transactions of the Leibnitz Society, and of the Royal Society of Sciences of Saxony[1]. The assumptions are,
>
> (...)
>
> From these axioms are deducible Ampère's laws of attraction of conductors, and those of Neumann and others, for the induction of currents. Here then is a really physical theory, satisfying the required conditions better perhaps than any yet invented, and put forth by

[1]When this was written, I [Maxwell] was not aware that part of M. Weber's Memoir is translated in Taylor's *Scientific Memoirs*, Vol. V. Art. XIV. The value of his researches, both experimental and theoretical, renders the study of his theory necessary to every electrician.

a philosopher whose experimental researches form an ample foundation for his mathematical investigations. What is the use then of imagining an electro-tonic state of which we have no distinctly physical conception, instead of a formula of attraction which we can readily understand? I would answer, that it is a good thing to have two ways of looking at a subject, and to admit that there *are* two ways of looking at it. Besides, I do not think that we have any right at present to understand the action of electricity, and I hold that the chief merit of a temporary theory is, that it shall guide experiment, without impeding the progress of the true theory when it appears. **There are also objections to making any ultimate forces in nature depend on the velocity of the bodies between which they act. If the forces in nature are to be reduced to forces acting between particles, the principle of the Conservation of Force [Energy] requires that these forces should be in the line joining the particles and functions of the distance only.** The experiments of M. Weber on the reverse polarity of diamagnetics, which have been recently repeated by Professor Tyndall, establish a fact which is equally a consequence of M. Weber's theory of electricity and of the theory of lines of force.

Along the same lines goes the introduction of Maxwell's main paper on electromagnetism of 1864, [24], *A dynamical theory of the electromagnetic field*, our emphasis in boldface:

PART I.

Introductory.

(1) The most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance according to mathematical laws.

In this way mathematical theories of statical electricity, of magnetism, of the mechanical action between conductors carrying currents, and of the induction of currents have been formed. In these theories the force acting between the two bodies is treated with reference only to the condition of the bodies and their relative position, and without any express consideration of the surrounding medium.

These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one another at a distance by attraction and repulsion. The most complete development of a theory of this kind is that of M. W. Weber[2], who has made the same theory include electrostatic and electromagnetic phenomena.

In doing so, however, he has found it necessary to assume that the force between two electric particles depends on their relative velocity, as well as on their distance.

This theory, as developed by MM. W. Weber and C. Neumann[3], is exceedingly ingenious, and wonderfully comprehensive in its application to the phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with the more authority, as it has served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent system of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown.

(2) The mechanical difficulties, however, which are involved in the assumption of particles acting at a distance with forces which depend on their velocities are such as to prevent me from considering this theory as an ultimate one, though it may have been, and may yet be useful in leading to the coordination of phenomena.

I have therefore preferred to seek an explanation of the fact in another direction, by supposing them to be produced by actions which go on in the surrounding medium as well as in the excited bodies, and endeavouring to explain the action between distant bodies without assuming the existence of forces capable of acting directly at sensible distances.

Helmholtz and Maxwell were wrong in stating that Weber's law did not comply with conservation of energy. For a simple proof of this conservation of energy in Weber's electrodynamics see, for instance: [25], [26] and [27], Chapter 6. But it was only after 1871 that Maxwell changed his mind. For instance, in article [853], page 484 of Vol. 2 of his *Treatise* he showed that Weber's electrodynamics was consistent with the principle of conservation of energy as it could

[2] "Electrodynamische Maassbestimmungen." *Leipzic Trans.* Vol. I. 1849, and Taylor's *Scientific Memoirs*, Vol. V. art. xiv.

[3] *Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur.* Halis Saxonum, 1858.

be derived from a velocity dependent potential energy. Helmholtz proof did not apply to Weber's force because this force depends not only on the distance and velocities of the charges but also on their acceleration, and this general case had not been considered by Helmholtz.

Weber succeeded in unifying electrodynamics and induction with electrostatics with his generalization of Coulomb's force. The next step would then be the unification of these interactions with gravitation. The natural path along Weber's procedure is to generalize even more Coulomb's law in order to derive gravitation. The forces of Ampère and Faraday were derived from second order terms and so the suspicion is that gravitation might be due to fourth order terms. And this leads us to our model to derive gravitation from electromagnetism, presented in the next Section.

3 Generalization of Weber's Law

The simplest generalization of Weber's potential energy, (15), is obtained with an expression given by:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{1}{r}\left[1 - \alpha\left(\frac{\dot{r}}{c}\right)^2 - \beta\left(\frac{\dot{r}}{c}\right)^4 - \gamma\left(\frac{\dot{r}}{c}\right)^6 - \ldots\right] . \tag{17}$$

Weber's potential energy is this expression with $\alpha = 1/2$, $\beta = \gamma = \ldots = 0$. Now we want to generalize this expression to include terms of fourth and higher orders in $1/c$. Some reasons have been stated above. Another one is related with Helmholtz criticism of Weber's law, [28], [29], Vol. 2, Chapter 23. In this work Helmholtz pointed out a negative mass behaviour which happens in Weber's electrodynamics when a test charge is located inside a charged spherical shell at high potential. Recently Phipps overcame this criticism proposing a generalization of Weber's potential energy given by:

$$U^P \equiv \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{1}{r}\sqrt{1 - \frac{\dot{r}^2}{c^2}} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{1}{r}\left(1 - \frac{\dot{r}^2}{2c^2} - \frac{\dot{r}^4}{8c^4} - \ldots\right) . \tag{18}$$

See, for instance: [30], [31] and [32]. This potential energy reduces to Weber's one for velocities small compared to c. This shows that one way of overcoming Helmholtz criticism is to modify Weber's energy including terms of the order $(\dot{r}/c)^4$. Yet another reason is related with the ultimate speed implied by Weber's theory: [33].

We could try other expressions for the energy including, for instance, odd powers in $\dot{r}/c$, higher derivatives of r (like $\ddot{r}$, d^3r/dt^3, ...), etc. But Eq. (17) seems to be simple and natural enough so that for the moment we stick with it. If it works and if we find the values of β, γ, etc. then we can try to discover the general expression from which (17) is a Taylor expansion.

The force is obtained from this expression by the usual procedure (5). This yields:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}}{r^2}\left(1 - \alpha\frac{\dot{r}^2 - 2r\ddot{r}}{c^2} - \beta\frac{\dot{r}^4 - 4\dot{r}^2 r\ddot{r}}{c^4} - \gamma\frac{\dot{r}^6 - 6\dot{r}^4 r\ddot{r}}{c^6} - ...\right) . \quad (19)$$

This is the basic expression to be utilized in this work.

4 Gravitation as an Interaction Between Neutral Dipoles

Our general procedure to derive gravitation from electromagnetism is to calculate the average force between two neutral dipoles using Eq. (19). Dipole 1 consists of charges q_{1+} and q_{1-} while in dipole 2 we have q_{2+} and q_{2-}. As usual each dipole is supposed to consist of a negative charge oscillating harmonically around the positive one. The positive charges of the dipoles are supposed to be located at the positions $\vec{R}_1 = x_1(t)\hat{x} + y_1(t)\hat{y} + z_1(t)\hat{z}$ and $\vec{R}_2 = x_2(t)\hat{x} + y_2(t)\hat{y} + z_2(t)\hat{z}$. In order to have a reasonable value for the force we perform an average of the nine cardinal cases (q_{1-} oscillating along the x, y or z axes, and the same for q_{2-}).

Our idea is that gravitation can be a statistical residual force between groups of neutral charges. As such we let each dipole have an arbitrary phase and then perform an average between all possible phase differences. Moreover, to the negative charge of each dipole is allowed a different amplitude (A_1, A_2) and frequency of oscillation (ω_1, ω_2). In order to ease the calculations we only impose that $\omega_1 = n\omega_2$, with $n = 1, 2, 3, ...$, although this is not essential for the result. So our model of a usual neutral dipole can be written as:

$$\left.\begin{array}{c} q_{1-} = -q_{1+} \text{ and } q_{2-} = -q_{2+} \ , \\ A_{1+} = A_{2+} = 0, \ A_{1-} \neq 0 \text{ and } A_{2-} \neq 0 \ . \end{array}\right\} \quad (20)$$

We utilize the dipoles as a representation of atomic systems so that the typical amplitudes of oscillation are of the order $A_1 \approx A_2 \approx 10^{-10}\ m$. Likewise, the typical frequencies of oscillation are in the range of infrared or microwave, so that $\omega_1 \approx \omega_2 \approx 10^{10}\ s^{-1}$. The typical period of oscillation is then of the short value of $T_1 \approx T_2 \approx 10^{-10}\ s$. We know that gravitation exists (that is, we observe its effects) in the man and earth scales, up to a galactic scale. So we usually consider the distance $R = |\vec{R}_1 - \vec{R}_2|$ between the dipoles spanning from a meter to $10^{20}\ m$. In these conditions:

$$\frac{A_1^2}{R^2} \ll 1, \ \frac{A_2^2}{R^2} \ll 1 \ , \quad (21)$$

$$\frac{\omega_1^2}{c^2} \gg \frac{1}{R^2}, \ \frac{\omega_2^2}{c^2} \gg \frac{1}{R^2} . \tag{22}$$

In order to give an idea of our procedure we present here explicitly a calculation of this force in a specific situation. We consider the two dipoles separated along the y axis and also the negative charges vibrating along this direction (later on we generalize these conditions). The positions of charges q_1 and q_2 can be written as

$$\left.\begin{array}{l} \vec{r}_1 = x_1(t)\hat{x} + [y_1(t) + A_1 \sin(\omega_1 t + \theta_1)]\,\hat{y} + z_1(t)\hat{z} \ , \\ \vec{r}_2 = x_1(t)\hat{x} + [y_2(t) + A_2 \sin(\omega_2 t + \theta_2)]\,\hat{y} + z_1(t)\hat{z} \ . \end{array}\right\} \tag{23}$$

These two charges, q_1 and q_2, can represent q_{1+}, q_{1-}, q_{2+} and q_{2-} utilizing (20).

We define the quantities

$$\begin{aligned} \vec{R} &= [x_1(t) - x_2(t)]\,\hat{x} + [y_1(t) - y_2(t)]\,\hat{y} + [z_1(t) - z_2(t)]\,\hat{z} \\ &= R_x\hat{x} + R_y\hat{y} + R_z\hat{z} \ , \end{aligned} \tag{24}$$

$$\vec{V} = \frac{d\vec{R}}{dt} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z} \ , \tag{25}$$

$$\vec{A} = \frac{d\vec{V}}{dt} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \ , \tag{26}$$

$$R = |\vec{R}|, \ V = |\vec{V}|, \ A = |\vec{A}| \ , \tag{27}$$

$$\dot{R} = \frac{dR}{dt}, \ \ddot{R} = \frac{d^2R}{dt^2} \ , \tag{28}$$

$$B_0 = A_1 \sin(\omega_1 t + \theta_1) - A_2 \sin(\omega_2 t + \theta_2) \ , \tag{29}$$

$$B_1 = A_1\omega_1 \cos(\omega_1 t + \theta_1) - A_2\omega_2 \cos(\omega_2 t + \theta_2) \ , \tag{30}$$

$$B_2 = -A_1\omega_1^2 \sin(\omega_1 t + \theta_1) + A_2\omega_2^2 \sin(\omega_2 t + \theta_2) \ . \tag{31}$$

We want to discuss here a typical gravitational problem, like the motion of the planets in the solar system. Taking the planet Mercury as an example and its interaction with the Sun we have $R \approx 6 \times 10^{10}\ m$, $V \approx 5 \times 10^4\ ms^{-1}$, $A \approx V^2/R \approx 4 \times 10^{-2}\ ms^{-2}$. This shows that if there is a relative motion between the dipoles we can assume that

$$\left.\begin{array}{l} R^2\omega_{1,2}^2 \gg V^2 \gg A_{1,2}^2\omega_{1,2}^2 \ , \\ R^2\omega_{1,2}^2 \gg RA \gg A_{1,2}^2\omega_{1,2}^2 \ . \end{array}\right\} \tag{32}$$

If there is no relative motion between the dipoles these approximations will not be necessary.

Utilizing (19) and (21) to (32) we can write the force exerted by q_2 on q_1 as (up to the sixth order in $1/c$):

$$\begin{aligned}
\vec{F}_{21} \quad = \quad & \frac{q_1 q_2}{4\pi\varepsilon_o} \frac{R_y \hat{y}}{|R_y^3|} \Big\{ 1 - \frac{\alpha}{c^2} \Big[3V_y^2 - 2\vec{V}\cdot\vec{V} - 2R_y A_y + 2V_y B_1 \\
& - 2A_y B_0 - 2R_y B_2 + B_1^2 - 2B_0 B_2 + 4A_y B_0 + 4B_0 B_2 \Big] \\
& - \frac{\beta}{c^4} \Big[V_y^4 - 4V_y^2(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) + 4V_y^3 B_1 - 4V_y^2 A_y B_0 \\
& - 4R_y V_y^2 B_2 - 8V_y(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) B_1 + 6V_y^2 B_1^2 \\
& - 4V_y^2 B_0 B_2 - 8V_y A_y B_0 B_1 - 8R_y V_y B_1 B_2 - 4(\vec{V}\cdot\vec{V} \\
& - V_y^2 + R_y A_y) B_1^2 + 4V_y B_1^3 - 8V_y B_0 B_1 B_2 - 4A_y B_0 B_1^2 \\
& - 4R_y B_1^2 B_2 + B_1^4 - 4B_0 B_1^2 B_2 + 8V_y^2 A_y B_0 + 8V_y^2 B_0 B_2 \\
& + 16V_y A_y B_0 B_1 + 16V_y B_0 B_1 B_2 + 8A_y B_0 B_1^2 + 8B_0 B_1^2 B_2 \Big] \\
& - \frac{\gamma}{c^6} \Big[V_y^6 - 6V_y^4(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) + 6V_y^5 B_1 \\
& - 6V_y^4 A_y B_0 - 6R_y V_y^4 B_2 - 24V_y^3(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) B_1 \\
& + 15V_y^4 B_1^2 - 6V_y^4 B_0 B_2 - 24V_y^3 A_y B_0 B_1 - 24R_y V_y^3 B_1 B_2 \\
& - 36V_y^2(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) B_1^2 + 20V_y^3 B_1^3 \\
& - 24V_y^3 B_0 B_1 B_2 - 36V_y^2 A_y B_0 B_1^2 - 36R_y V_y^2 B_1^2 B_2 \\
& - 24V_y(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) B_1^3 + 15V_y^2 B_1^4 \\
& - 36V_y^2 B_0 B_1^2 B_2 - 24V_y A_y B_0 B_1^3 - 24R_y V_y B_1^3 B_2 \\
& - 6(\vec{V}\cdot\vec{V} - V_y^2 + R_y A_y) B_1^4 + 6V_y B_1^5 - 24V_y B_0 B_1^3 B_2 \\
& - 6A_y B_0 B_1^4 - 6R_y B_1^4 B_2 + B_1^6 - 6B_0 B_1^4 B_2 + 12V_y^4 A_y B_0 \\
& + 12V_y^4 B_0 B_2 + 48V_y^3 A_y B_0 B_1 + 48V_y^3 B_0 B_1 B_2 \\
& + 72V_y^2 A_y B_0 B_1^2 + 72V_y^2 B_0 B_1^2 B_2 + 48V_y A_y B_0 B_1^3 \\
& + 48V_y B_0 B_1^3 B_2 + 12A_y B_0 B_1^4 + 12B_0 B_1^4 B_2 \Big] \Big\} \ . \qquad (33)
\end{aligned}$$

We now calculate the force between the dipoles using Eqs. (20), (33) and adding the four pair of forces, namely ($\vec{F}_{ji}$ is the force exerted by j on i):

$$\vec{F} = \vec{F}_{2+,1+} + \vec{F}_{2+,1-} + \vec{F}_{2-,1+} + \vec{F}_{2-,1-} \ . \qquad (34)$$

After such a sum we will get the only result which interests us here, the average value. For this end we perform the averages on the phases and the average on time:

$$< \vec{F} > = \frac{1}{T_2} \int_0^{T_2} dt \frac{1}{2\pi} \int_0^{2\pi} d\theta_1 \frac{1}{2\pi} \int_0^{2\pi} d\theta_2 \vec{F} \ . \qquad (35)$$

Observing as above that $T_2 \approx 10^{-10}\ s$ we can consider $\vec{R}$, $\vec{V}$ and $\vec{A}$ as constants during this short time interval. Performing this average indicated in (35) for the resultant force exerted by dipole 2 on dipole 1 yields (see (34), (33) and (20)):

$$< \vec{F} >= \frac{q_{1+}q_{2+}}{4\pi\varepsilon_o}\frac{R_y\hat{y}}{|R_y|^3}\frac{A_{1-}^2\omega_1^2 A_{2-}^2\omega_2^2}{2c^4}\left[\beta - \gamma\frac{27v_y^2 - 18(\vec{v}\cdot\vec{v} + R_yA_y)}{c^2}\right] \ . \quad (36)$$

We now follow the same procedure to calculate the force of dipole 2 on 1 in the other cardinal situations. When the dipoles are separated along the y axis and both negative charges oscillate along the x direction, we have:

$$\left.\begin{aligned} \vec{r}_1 &= [x_1(t) + A_1\sin(\omega_1 t + \theta_1)]\,\hat{x} + y_1(t)\hat{y} + z_1(t)\hat{z}\ , \\ \vec{r}_2 &= [x_1(t) + A_2\sin(\omega_2 t + \theta_2)]\,\hat{x} + y_2(t)\hat{y} + z_1(t)\hat{z}\ . \end{aligned}\right\} \quad (37)$$

Utilizing (19), (21), (22), (37) and (24) to (32) we can write the force exerted by q_2 on q_1 as (up to the sixth order in $1/c$):

$$\begin{aligned} \vec{F}_{21} \ = \ & \frac{q_1q_2}{4\pi\varepsilon_o}\frac{\vec{R}}{R^3}\left\{1 - \frac{\alpha}{c^2}\left[\frac{(\vec{R}\cdot\vec{V})^2}{R^2} - 2R\ddot{R} - 4V_xB_1 - 2A_xB_o - 2B_1^2\right.\right. \\ & \left. - 2B_oB_2\right] - \frac{\beta}{c^4}\left[\frac{(\vec{R}\cdot\vec{V})^4}{R^4} - \frac{4(\vec{R}\cdot\vec{V})^2}{R^2}(R\ddot{R} + 2V_xB_1 + A_xB_o + B_1^2\right. \\ & \left. + B_oB_2) - \frac{8(\vec{R}\cdot\vec{V})V_xB_oR\ddot{R}}{R^2}\right] - \frac{\gamma}{c^6}\left[\frac{(\vec{R}\cdot\vec{V})^6}{R^6} - \frac{6(\vec{R}\cdot\vec{V})^4}{R^4}(R\ddot{R}\right. \\ & \left.\left. + 2V_xB_1 + A_xB_o + B_1^2 + B_oB_2) - \frac{24(\vec{R}\cdot\vec{V})^3V_xB_oR\ddot{R}}{R^4}\right]\right\} \\ & + \frac{q_1q_2}{4\pi\varepsilon_o}\frac{\hat{x}}{R^2}\left[\frac{2\alpha\ddot{R}B_o}{c^2} + \frac{4\beta(\vec{R}\cdot\vec{V})^2\ddot{R}B_o}{c^4R^2} + \frac{6\gamma(\vec{R}\cdot\vec{V})^4\ddot{R}B_o}{c^6R^4}\right] \ . \end{aligned} \quad (38)$$

Utilizing (20), (34) and (35) in this equation yields a zero average value. By symmetry the same happens when both dipoles are separated along the y axis and the negative charges of both dipoles oscillate along the z axis.

Another situation is the following: The dipoles separated along the y axis while q_{1-} oscillates along the x axis and q_{2-} along the z axis:

$$\left.\begin{aligned} \vec{r}_1 &= [x_1(t) + A_1\sin(\omega_1 t + \theta_1)]\,\hat{x} + y_1(t)\hat{y} + z_1(t)\hat{z}\ , \\ \vec{r}_2 &= x_1(t)\hat{x} + y_2(t)\hat{y} + [z_1(t) + A_2\sin(\omega_2 t + \theta_2)]\,\hat{z}\ . \end{aligned}\right\} \quad (39)$$

Yet in another situation we have both dipoles separated along the y axis with q_{1-} oscillating along the z axis and q_{2-} along the x axis. Utilizing (19) to

(22), (24) to (32), (34) and (35) yields a zero average value for these two last situations as well.

On the other hand when the dipoles are separated along the y axis and q_{1-} oscillates along this axis while q_{2-} oscillates along the x axis, we have:

$$\left.\begin{array}{l}\vec{r}_1 = x_1(t)\hat{x} + [y_1(t) + A_1\sin(\omega_1 t + \theta_1)]\,\hat{y} + z_1(t)\hat{z}\ , \\ \vec{r}_2 = [x_1(t) + A_2\sin(\omega_2 t + \theta_2)]\,\hat{x} + y_2(t)\hat{y} + z_1(t)\hat{z}\ .\end{array}\right\} \quad (40)$$

Following the same procedure as above the resultant average value for this situations is given by:

$$< \vec{F} >= -\frac{q_{1+}q_{2+}}{4\pi\varepsilon_o}\frac{R_y\hat{y}}{|R_y|^3}\frac{A_{1-}^2\omega_1^2 A_{2-}^2\omega_2^2}{c^4}\left(\beta + \frac{9\gamma A_{1-}^2 w_1^2}{8c^2}\right) . \quad (41)$$

The same result is obtained when q_{1-} oscillates along the y axis while q_{2-} oscillates along the z axis.

When q_{1-} oscillates along the x or z axis, while q_{2-} oscillates along the y axis, being both dipoles separated along the y axis, the same procedure yields:

$$< \vec{F} >= -\frac{q_{1+}q_{2+}}{4\pi\varepsilon_o}\frac{R_y\hat{y}}{|R_y|^3}\frac{A_{1-}^2\omega_1^2 A_{2-}^2\omega_2^2}{c^4}\left(\beta + \frac{9\gamma A_{2-}^2 w_2^2}{8c^2}\right) . \quad (42)$$

The final average result is found taking the average of these nine cases and generalizing for any relative position in space (and not for separation between the dipoles only along the y axis). The value for the resultant average force of dipole 2 on dipole 1 is:

$$\vec{F} = -\frac{7\beta}{18}\frac{q_{1+}q_{2+}}{4\pi\varepsilon_o}\frac{\vec{R}}{R^3}\frac{A_{1-}^2\omega_1^2 A_{2-}^2\omega_2^2}{c^4}\left(1 + \frac{18\gamma}{7\beta}\frac{2.5\dot{R}^2 - R\ddot{R}}{c^2}\right) . \quad (43)$$

In this expression $\dot{R} = dR/dt = \vec{R}\cdot\vec{V}/R$ and $\ddot{R} = d\dot{R}/dt = (1/R)(\vec{V}\cdot\vec{V} + \vec{R}\cdot\vec{A} - \dot{R}^2)$.

This is the most important result of this work. It indicates the existence of a non zero resultant force between two neutral dipoles due to terms of fourth and higher orders of $1/c$ in Eq. (19).

We first concentrate on the fourth order term in Eq. (43). We observe that if $\beta > 0$ there will be an attractive force between the dipoles which falls as $1/R^2$ and is directed along the line joining them. But these are exactly the properties of Newton's law of gravitation, Eq. (2). So we can interpret Eq. (43) as a derivation of gravitation from electromagnetism. If in $\vec{R}_1$ there are N_1 dipoles and in $\vec{R}_2$ there are N_2 dipoles we can say that each group has a "mass" m_1 and m_2 which we identify through the equation below

$$\frac{7\beta}{18}\frac{N_1 q_{1+}N_2 q_{2+}}{4\pi\varepsilon_o}\frac{A_{1-}^2\omega_1^2 A_{2-}^2\omega_2^2}{c^4} = Gm_1m_2\ . \quad (44)$$

The major evidence supporting our performing such an identification is the correct order of magnitude we have. For instance, supposing $\beta \approx 1$, $N_1 = N_2 = 1$, $q_{1+} = q_{2+} = e$ (where $-e$ is the charge of the electron) and the previous values of the amplitudes and frequencies we obtain for the left side of Eq. (44) the approximate value of $10^{-62}\ Nm^2$. This means that in this case we would have "masses" $m_1 = m_2 \approx 10^{-26}\ kg$, which is of the same order of magnitude as the mass of the hydrogen atom. Obviously we cannot obtain exact numbers due to uncertainties in the values of A_{1-}, A_{2-}, ω_1, ω_2 and also because we do not have a precise value of β. But it is amazing that we could obtain the correct order of magnitude in this simple model, namely, the gravitational force being approximately 10^{-40} times smaller than the electrostatic force at the same distance.

With Eq. (44) in Eq. (43) we can reproduce exactly the same results of our previous works on gravitation (quantitative implementation of Mach's principle, derivation of the proportionality between inertial and gravitational masses, calculation of the observed precession of the perihelion of the planets, etc.), provided that $\gamma/\beta < 0$: See [34], [35], [36], [37] and [38]. This means $\gamma < 0$, as we fixed previously $\beta > 0$. So even the inertia of a body can be seen as a sixth order electromagnetic effect. In order to get the correct precession of the perihelion of the planets using Eqs. (43) and (44) we need to impose a more restrictive constraint. The calculation is a standard one, [34], and the correct result ($\triangle\varphi = 6\pi GM/(c^2a(1-\epsilon^2))$), can be obtained through Eq. (43) with $\gamma/\beta = -7/3$. This is a relevant result as it helps to fix the values of β and γ.

In conclusion we can say that with this preliminary model we tried to show the possibility of deriving gravitation from electromagnetism, even with the correct orders of magnitude. This is a constructive model in which we utilized only electromagnetism to show that a neutral dipole can exert an attractive force on another neutral dipole.

We restricted ourselves to the sixth order but it is reasonable to suspect that there will be present other effects at the eight and higher orders. The study of these effects requires a much larger analysis which is outside the scope of this work.

We hope this model can cast some light on the unification of the forces of nature.

Acknowledgements

The author wishes to thank CNPq, FAPESP and FAEP-UNICAMP (Brazil) for financial support during the past few years.

References

[1] E. Mach. *The Science of Mechanics — A Critical and Historical Account of Its Development.* Open Court, La Salle, 1960.

[2] A. K. T. Assis. *Weber's Electrodynamics.* Kluwer Academic Publishers, Dordrecht, 1994.

[3] P. Heering. On Coulomb's inverse square law. *American Journal of Physics*, 60:988–994, 1992.

[4] P. Graneau. *Ampere-Neumann Electrodynamics of Metals.* Hadronic Press, Palm Harbor, 2nd edition, 1994.

[5] P. Graneau and P. N. Graneau. *Newton Versus Einstein — How Matter Interacts with Matter.* Carlton Press, New York, 1993.

[6] W. Weber. On the measurement of electro-dynamic forces. In R. Taylor, editor, *Scientific Memoirs,* Vol. 5, pages 489–529, New York, 1966. Johnson Reprint Corporation.

[7] W. Weber. On the measurement of electric resistance according to an absolute standard. *Philosophical Magazine*, 22:226–240 and 261–269, 1861.

[8] W. Weber. Electrodynamic measurements — Sixth memoir, relating specially to the principle of the conservation of energy. *Philosophical Magazine*, 43:1–20 and 119–149, 1872.

[9] W. Weber. *Wilhelm Weber's Werke,* W. Voigt, (ed.), volume 1, *Akustik, Mechanik, Optik und Wärmelehre.* Springer, Berlin, 1892.

[10] W. Weber. *Wilhelm Weber's Werke,* E. Riecke (ed.), volume 2, *Magnetismus.* Springer, Berlin, 1892.

[11] W. Weber. *Wilhelm Weber's Werke, H. Weber (ed.),* volume 3, *Galvanismus und Elektrodynamik,* first part. Springer, Berlin, 1893.

[12] W. Weber. *Wilhelm Weber's Werke,* H. Weber, (ed.), volume 4, *Galvanismus und Elektrodynamik,* second part. Springer, Berlin, 1894.

[13] W. Weber and E. H. Weber. *Wilhelm Weber's Werke,* E. Riecke (ed.), volume 5, *Wellenlehre auf Experimente gegründet oder über die Wellen tropfbarer Flüssigkeiten mit Anwendung auf die Schall- und Lichtwellen.* Springer, Berlin, 1893.

[14] W. Weber. *Wilhelm Weber's Werke,* F. Merkel and O. Fischer (eds.), volume 6, *Mechanik der Menschlichen Gehwerkzeuge.* Springer, Berlin, 1894.

[15] F. Kirchner. Determination of the velocity of light from electromagnetic measurements according to W. Weber and R. Kohlrausch. *American Journal of Physics*, 25:623–629, 1957.

[16] L. Rosenfeld. The velocity of light and the evolution of electrodynamics. *Il Nuovo Cimento*, Supplement to Vol. 4:1630–1669, 1957.

[17] L. Rosenfeld. Kirchhoff, Gustav Robert. In C. C. Gillispie, editor, *Dictionary of Scientific Biography*, Vol. 7, pages 379–383, New York, 1973. Scribner.

[18] C. Jungnickel and R. McCormmach. *Intellectual Mastery of Nature — Theoretical Physics from Ohm to Einstein*, volume 1–2. University of Chicago Press, Chicago, 1986.

[19] G. Kirchhoff. On a deduction of Ohm's law in connexion with the theory of electrostatics. *Philosophical Magazine*, 37:463–468, 1850.

[20] G. Kirchhoff. On the motion of electricity in wires. *Philosophical Magazine*, 13:393–412, 1857.

[21] P. Graneau and A. K. T. Assis. Kirchhoff on the motion of electricity in conductors. *Apeiron*, 19:19–25, 1994.

[22] H. von Helmholtz. On the conservation of force; a physical memoir. In J. Tyndall and W. Francis, editors, *Scientific Memoirs*, Vol. 7, pages 114–162, New York, 1966. Johnson Reprint Corporation.

[23] J. C. Maxwell. On Faraday's lines of force. In W. D. Niven, editor, *The Scientific Papers of James Clerk Maxwell*, pages 155–229 (Vol. 1), New York, 1965. Dover. Article originally published in 1855.

[24] J. C. Maxwell. A dynamical theory of the electromagnetic field. In W. D. Niven, editor, *The Scientific Papers of James Clerk Maxwell*, pages 526–597 (Vol. 1), New York, 1965. Dover. Article originally published in 1864.

[25] J. P. Wesley. Weber electrodynamics extended to include radiation. *Speculations in Science and Technology*, 10:47–61, 1987.

[26] J. P. Wesley. Weber electrodynamics, Part I. General theory, steady current effects. *Foundations of Physics Letters*, 3:443–469, 1990.

[27] J. P. Wesley. *Selected Topics in Advanced Fundamental Physics*. Benjamin Wesley Publisher, Blumberg, 1991.

[28] H. von Helmholtz. On the theory of electrodynamics. *Philosophical Magazine*, 44:530–537, 1872.

[29] J. C. Maxwell. *A Treatise on Electricity and Magnetism*, volume 1–2. Dover, New York, 1954.

[30] T. E. Phipps Jr. Toward modernization of Weber's force law. *Physics Essays*, 3:414–420, 1990.

[31] T. E. Phipps Jr. Weber-type laws of action-at-a-distance in modern physics. *Apeiron*, 8:8–14, 1990.

[32] T. E. Phipps Jr. Derivation of a modernized Weber force law. *Physics Essays*, 5:425–428, 1992.

[33] A. K. T. Assis and J. J. Caluzi. A limitation of Weber's law. *Physics Letters A*, 160:25–30, 1991.

[34] A. K. T. Assis. On Mach's principle. *Foundations of Physics Letters*, 2:301–318, 1989.

[35] A. K. T. Assis. On the absorption of gravity. *Apeiron*, 13:3–11, 1992.

[36] A. K. T. Assis. Deriving gravitation from electromagnetism. *Canadian Journal of Physics*, 70:330–340, 1992.

[37] A. K. T. Assis. Compliance of a Weber's force law for gravitation with Mach's principle. In P. N. Kropotkin et al., editor, *Space and Time Problems in Modern Natural Science, Part II*, pages 263–270, St.-Petersburg, 1993. Tomsk Scientific Center of the Russian Academy of Sciences. Series: "The Universe Investigation Problems," Issue 16.

[38] A. K. T. Assis. A steady-state cosmology. In H. C. Arp, C. R. Keys, and K. Rudnicki, editors, *Progress in New Cosmologies: Beyond the Big Bang*, pages 153–167, New York, 1993. Plenum Press.